ON THE SYSTEMATIC GEOMETRICAL VARIATION OF SHIP FORMS

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Summary

In connection with the derivation of the lines for a new design from those of a similar basis ship, consideration has been given to the well-known "one minus prismatic" method of correcting for the difference in fineness between the two forms and the limitations inherent in this method have been discussed and enumerated. Principal among these is the fact that the fineness and the extent of the parallel middle body cannot be varied independently. To overcome this and other consequent restrictions more general methods of form derivation have been developed which permit independent variation of not only the fineness and LCB position, but also the extent of parallel middle in both the fore and after bodies (or alternatively the lengths of entrance and run). There are limits to which the fineness can be varied by these means, but it is shown that the range is sufficiently wide to cover practical requirements. The methods can be applied to any existing parent form and, apart from the preliminary calculation of the necessary geometrical particulars of the form, they are just as convenient to apply as the usual "one minus prismatic" variation.

It is suggested that these more general methods of form derivation might be found useful in connection with the following:—

(i) Making systematic variations to parent forms for methodical series of resistance experiments, as there is a free choice of the more important geometrical features likely to influence the resistance.

(ii) Preparing body sections for new designs and at least for those cases where, for reasons explained in the paper, the usual "one minus prismatic" variation cannot be applied.

The methods described have been used to derive the forms for methodical series of resistance experiments on ship models carried out for the British Shipbuilding Research Association. Several illustrated examples showing typical systematic variations to actual area curves are shown in Figs. 10 to 14 inclusive. There are three appendices, as follows:—

Appendix I.—Several detailed worked examples are given which might be found helpful to the user in a design office and particularly the form of calculation for the general case given in Example G. Examples A and B may also be of particular interest in that they give a treatment of the "one minus prismatic" variation in which the LCB position is accurately controlled without the use of empirical data.

Appendix II.—This gives the mathematical derivation of the formulae quoted in the paper.

Appendix III.—For the sake of completeness a brief account is given of the well-known method of changing the position of the LCB in a parent form by "swinging" the area curve.

1. Introduction

In deriving the lines for a new design from a similar basis ship, it is usual to correct for fineness by adjusting the longitudinal spacing of the transverse sections to suit the new curve of areas. In this connection the practice in use in many design offices is to make the revised spacing of the sections from the ends of the ship proportional to the difference between the respective prismatic coefficients and unity. This well-known "one minus prismatic" correction is very useful and convenient to apply, as the new body sections can be lifted directly from the half-breadth plan of the basis ship, subject of course to proportional expansion or contraction of the offsets to suit the beam of the new design. The method has its limitations, however, and these have been dealt with in detail in the next section of the paper. Principal among these is the fact that the fineness of the form as a whole is adjusted simply by inserting or removing parallel middle body from the basis ship and contracting or expanding the entrance and run as necessary. This means that the fineness of the entrance and run remain the same in the basis and derived forms and that the extent of parallel middle body cannot be varied independently of the prismatic coefficient.

The need arose recently for a more general system of form derivation and methods have been developed in the paper whereby the fullness of the entrance and run can be systematically adjusted, which permits the total prismatic coefficient and extent of parallel middle body to be varied independently. Thus, when considering the systematic variation of a parent form, numerous possibilities now present themselves and the relations have been worked out for several special cases which might be found useful in practice. The relations for the general case have also been given which cover all the special cases considered. There are limits to which the fullness can be varied by these means, but it will be seen that the range available is sufficient to cover any variation likely to be required in practice. The relative fullness of the fore and after bodies as affecting the longitudinal position of the centre of buoyancy has also been dealt with and means given whereby allowance for this can be made in the various cases considered. These more general

methods of form derivation can be applied to any existing ship form and, apart from the preliminary calculation of the necessary geometrical particulars of the form, they are just as convenient to apply as the usual "one minus prismatic" variation. For convenience, the relations given in the paper are in terms of geometrical properties of the fore and after bodies and it is therefore not necessary to consider the entrance and run separately.

It is suggested that these more general methods of form derivation might be considered suitable for making systematic variations to parent forms in connection with methodical series of resistance experiments as they permit *independent* variation of the more important geometrical features likely to influence the resistance, viz. prismatic coefficient, LCB position, and the extent of parallel middle in both the fore and after bodies (or alternatively the lengths of entrance and run). It is also suggested that the methods might be used with advantage in design offices in preparing body sections for new designs and at least for those cases where, for reasons explained in the next section, the usual "one minus prismatic" variation cannot be applied.

With regard to the practical application of the methods in a design office, attention is drawn particularly to Appendix I where several detailed worked examples are given and especially to the General Case G (page 18). The practical essence of the methods of form derivation developed in the paper is given in this example and the relations used may be considered to be in the simplest form for everyday use. Attention is also directed to worked examples A and B which give what may be found to be a useful treatment of the "one minus prismatic" method of form variation in which the LCB position is accurately controlled without recourse to empirical data.

The mathematical derivation of the formulae quoted in the paper is given in Appendix II.

For the sake of completeness a brief account is given in Appendix III of the well-known method of changing the LCB position of a parent form by "swinging" the curve of sectional areas. As will be seen, however, in this method there is again no control over the longitudinal position of the parallel middle body (or maximum section) in the derived form. This is displaced bodily either forward or aft according to the required movement of the LCB.

2. The "One Minus Prismatic" Variation

Referring to Fig. 1, the curve A B C represents the curve of areas of the basis ship for one half of the body. It is convenient to consider this half-body as being one unit long and the maximum ordinate of the area curve also equal to unity. All horizontal dimensions are therefore fractions of the half length and the area under the curve A B C is numerically equal to the prismatic coefficient of the half-body.

For the basis ship:—

ϕ = the prismatic coefficient of the half-body.

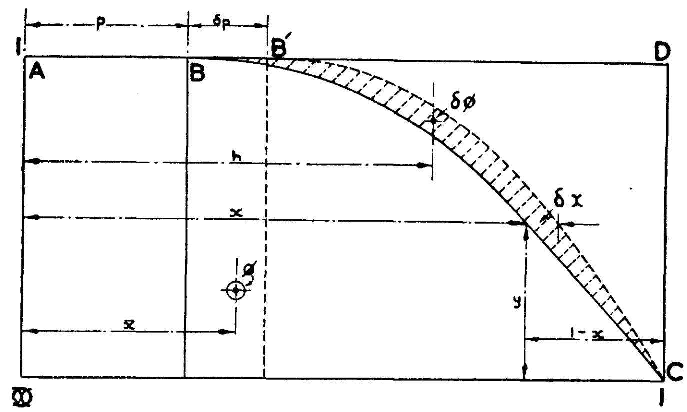

FIG. 1

$\bar{x}$ = the fractional distance from midships of the centroid of the half-body.

p = the fractional parallel middle of the half-body.

x = the fractional distance of any transverse section from midships.

y = the area of the transverse section at x expressed as a fraction of the maximum ordinate.

For the derived form:—

$\delta\phi$ = the required change in prismatic coefficient of the half-body.

δp = the *consequent* change in parallel middle body.

δx = the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient.

h = the fractional distance from midships of the centroid of the added "sliver" of area represented by $\delta\phi$.

A B′ C is the curve of areas of the derived form having the required prismatic coefficient of $\phi + \delta\phi$.

In accordance with the usual practice the new spacing of the sections from the end of the body is made proportional to the difference between the respective prismatics and unity;

i.e. $$\frac{1-(x+\delta x)}{1-x} = \frac{1-(\phi+\delta\phi)}{1-\phi}$$

Therefore $$\frac{\delta x}{1-x} = \frac{\delta\phi}{1-\phi}$$

or $$\delta x = \frac{\delta\phi}{1-\phi}(1-x) \quad . \quad . \quad . \quad . \quad (1)$$

The area B C D is clearly equal to $(1-\phi)$ and the above variation simply amounts to reducing this in the ratio $\dfrac{1-(\phi+\delta\phi)}{(1-\phi)}$.

Hence the new area B′ C D $= 1-(\phi+\delta\phi)$, and therefore the new prismatic coefficient $= \phi + \delta\phi$, which proves the method. The change in the parallel middle body δp will equal δx at $x = p$.

Therefore $$\delta p = \frac{\delta\phi}{1-\phi}(1-p) \quad . \quad . \quad . \quad . \quad (2)$$

or
$$\frac{\delta p}{(1-p)} = \frac{\delta \phi}{(1-\phi)}$$

It will thus be seen that the change in fineness of the form produces a definite change in the parallel middle body and the latter cannot be varied independently by this particular method. The change in fineness is brought about simply by inserting or removing parallel middle body and contracting or expanding the entrance or run as necessary. In other words the prismatic coefficients of the entrance and run remain the same in the basis and the derived forms.

Although, therefore, the above process is useful and simple to apply, it would appear to have the following disadvantages:—

(i) There is no control over the extent of the parallel middle body in the derived forms, i.e. p and ϕ cannot be varied independently.

(ii) The process cannot be applied to reduce the fullness of a basis ship which has no parallel middle body.

(iii) A basis form having no parallel middle body cannot be increased in fullness without the introduction of parallel middle.

(iv) The prismatic coefficient of the entrance or run cannot be adjusted.

(v) For a given change in fullness the longitudinal distribution of the displacement added (or removed) cannot be controlled; the maximum longitudinal shift of sections is restricted to the "shoulders" of the area curve.

For the above reasons the method is not entirely suitable for developing the forms of a methodical series of models as there is not a free choice of the various factors likely to influence the resistance. It is only *one particular* method of deriving forms of varying fullness from a basis ship.

In applying this method, if particular regard is to be paid to the position of the LCB in the derived form, the relative fineness of the fore and after bodies must be suitably adjusted and in practice the amount of adjustment necessary to produce a given shift of LCB is usually based on experience with similar forms. For an exact assessment of the necessary adjustment in fineness of the fore and after bodies it is necessary to know the lever h of the centroid of the added sliver of area in both bodies. It may not be generally known that this lever can be readily calculated from the geometrical properties of the original basis curve of areas A B C. For example, as shown in Appendix II, the exact value of h is given by

$$h = \frac{\phi(1-2\bar{x})}{1-\phi} + \frac{\delta\phi}{2(1-\phi)^2}\left[1-2\phi(1-\bar{x})\right] \quad (3)$$

where $\bar{x}$ is the centroid of the original area A B C from midships. For moderate changes in ϕ, experience shows that the second term is negligible compared with the first and a very good first approximation to h is given simply by:—

$$h = \frac{\phi(1-2\bar{x})}{1-\phi} \quad . \quad . \quad . \quad . \quad (4)$$

A case arose recently of a rather fine parent form* in which there was no parallel middle and it was desired to vary the fullness in a systematic manner over a definite range without the introduction of parallel middle body. This amounted to varying the fullness of the entrance and run and a method of doing this has been developed which is described in the next section.

3. Varying the Fullness of an Entrance or Run not Associated with Parallel Middle Body

Referring to Fig. 2 the various items have the same significance as in Fig. 1; it will be noted, however, that there is no parallel middle body in either the basis or

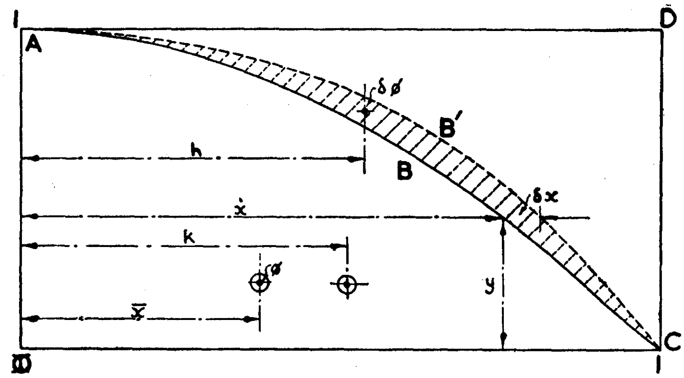

FIG. 2

derived form. In other words the length of entrance (or run) is equal to the length of the half-body and the variation amounts to varying the fullness of the entrance (or run). The required terminal conditions are obtained by using a relation between δx and x of the form

$$\delta x = c \,.\, x(1-x)$$

where c is a constant.

As shown in Appendix II, in order to increase the prismatic coefficient of the half-body by $\delta\phi$ the required relation is:—

$$\delta x = \frac{\delta\phi \,.\, x(1-x)}{\phi(1-2\bar{x})} \quad . \quad . \quad . \quad . \quad (5)$$

It will be noted that $\delta x = 0$ at both $x = 1$ and $x = 0$. It can be shown that δx is a maximum at $x = \frac{1}{2}$. Thus the curve is filled out generally and there is no concentration of added area either towards A or C.

The first approximation to the lever h is given by

$$h = \frac{2\bar{x} - 3k^2}{1-2\bar{x}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where k is the radius of gyration (or lever of the second moment) of the original curve A B C about midships and, as before, $\bar{x}$ is the distance of the centroid (or lever of the first moment) of the original curve from midships. It can be shown that the exact lever h is given by

$$h = \frac{2\bar{x} - 3k^2}{1-2\bar{x}} + \frac{\delta\phi}{\phi}\,\frac{(\bar{x} - 3k^2 + 2r^3)}{(1-2\bar{x})^2} \quad . \quad (7)$$

* B.S.R.A. Trawler Series.

where r = the lever of the third moment of the original curve A B C about midships. Here again, however, experience shows that the second term can generally be neglected.

It will be noted in the expression for δx that, as in the "one minus prismatic" variation, the only variable used is x which represents the longitudinal spacing of the transverse sections and that the ordinate y of the area curve is not involved. The only other items in the expression are all constant in a given case and refer only to the geometrical properties of the original parent curve. This is particularly convenient as it enables the body sections for the derived form to be lifted directly from the half-breadth plan of the basis ship at the revised section spacing given by δx. There is thus no need to plot the new area curve or draw out the new water-lines. This implies that sections in the basis and derived forms having the same areas will have identical shapes. If the beam and draught of the new form are to be different from the basis ship the half-breadths of the derived sections can be proportionally expanded or contracted by any convenient means. In these respects the proposed method of form derivation resembles the usual "one minus prismatic" variation discussed in the previous section and is just as convenient to apply.

There are definite limits, however, to which the fullness of a given form can be varied by this method. In this connection it can be shown that the absolute limit of fullness is given by:—

$$\delta \phi = \pm \phi (1 - 2 \bar{x}) \quad . \quad . \quad . \quad . \quad (8)$$

At $\delta \phi = - \phi (1 - 2 \bar{x})$ an angular shoulder is generally formed at A and at $\delta \phi = + \phi (1 - 2 \bar{x})$ the derived area curve at C approaches the base line either vertically or at a very steep angle according as to whether the slope of the original curve at C is finite or zero respectively. In Fig. 10 the forebody parent curve of the B.S.R.A. Trawler Series is shown which has been filled out and fined to the above limits using the present method; this form has no parallel middle body and the forebody is identical with the entrance. In Fig. 11 the entrance curve of a merchant ship form having a block coefficient of 0·70 has been treated in a similar manner. The prismatic coefficient of the entrance of the Trawler Series is about 0·637 and, as it is not associated with parallel middle body, the area curve at midships falls away rather quickly. In this case it will be noted that derived curves of good shape are obtained when the variation in ϕ is taken at least half way to the upper and lower limits, i.e. when $\delta \phi = \pm \frac{1}{2} \phi (1 - 2 \bar{x})$. This represents a variation of $\pm 0{\cdot}096$ on 0·637 which is about $\pm$ 15 per cent on the prismatic coefficient of the entrance. With regard to the parent curve shown in Fig. 11 for which the prismatic coefficient of the entrance is 0·656, this was actually associated with parallel middle body and the fall of the curve in way of the shoulder is more gradual as a consequence. It will be noted here that good derived curves are obtained when the variation in ϕ_e is taken up to about $\frac{3}{4}$ of the upper and lower limits. This represents a variation of $\pm 0{\cdot}142$ on 0·656 which is about $\pm$ 22 per cent on the prismatic coefficient of the entrance. It is considered that, for a given basic form, this range would include any practical variation in fullness of the entrance or run likely to be required. As a general guide then, it would be well on the safe side to consider a practical limit of

$$\delta \phi = \pm \tfrac{1}{2} \phi (1 - 2 \bar{x}) \quad . \quad . \quad . \quad (9)$$

This applies only to the present case, of course (Fig. 2), where the entrance or run is identical with the fore or after bodies respectively. This systematic variation has been used in deriving the forms for the B.S.R.A. Methodical Series of model experiments on the resistance of Trawlers.

This method can be extended to vary the fullness of the entrance or run when associated with parallel middle body and has the advantage that both the parallel middle body and the prismatic coefficient for the entrance or run can be varied *independently*. Thus when considering the systematic variation in fullness of a parent form numerous possibilities present themselves. In the following section 4 some special cases involving parallel middle body are considered and in section 5 the relations for the general case are given, which also covers the usual "one minus prismatic" variation discussed in section 2.

4. Some Special Cases involving Parallel Middle Body

(*a*) **Basis Form:** *No parallel middle in the half-body, i.e. $p = o$.*

Derived Form: *Required to introduce parallel middle body equal to δp keeping ϕ constant.*

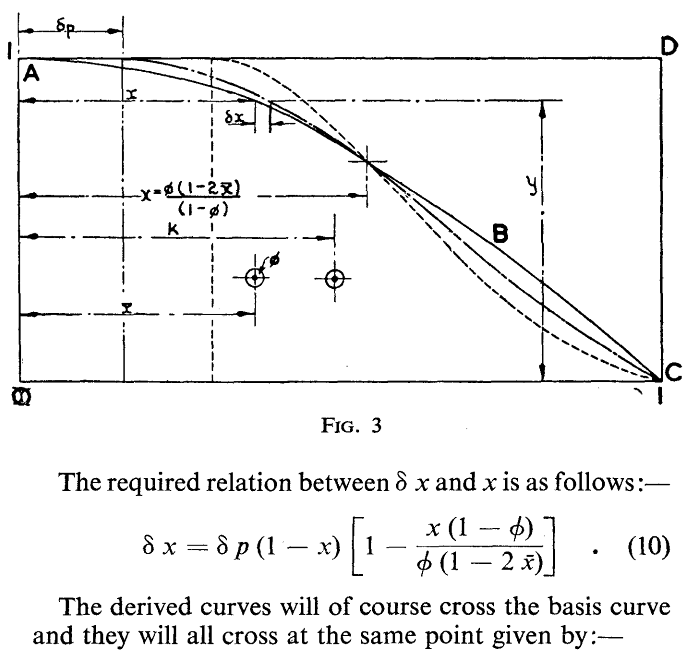

FIG. 3

The required relation between δx and x is as follows:—

$$\delta x = \delta p (1 - x) \left[1 - \frac{x (1 - \phi)}{\phi (1 - 2 \bar{x})} \right] \quad . \quad (10)$$

The derived curves will of course cross the basis curve and they will all cross at the same point given by:—

$$x = \frac{\phi (1 - 2 \bar{x})}{(1 - \phi)} \quad . \quad . \quad . \quad . \quad (11)$$

As no area is added to the basis curve A B C, the lever h is indeterminate, but the change in the longitudinal

movement of $\bar{x}$, the centroid of the original area, is given by:—

$$\delta \bar{x} = -\delta p\left[\frac{(1-\phi)(2\bar{x}-3k^2)}{\phi(1-2\bar{x})} - (1-2\bar{x})\right] \quad \text{(first approx.)} \quad . \quad (12)$$

The absolute limit to which parallel middle could be introduced is given by the relation:—

$$\delta p = \frac{\phi(1-2\bar{x})}{(1-2\phi \,.\, \bar{x})} \quad . \quad . \quad . \quad . \quad (13)$$

The corresponding practical limit is:—

$$\delta p = \frac{\phi(1-2\bar{x})}{2-\phi(1+2\bar{x})} \quad \text{(approx.)} \quad . \quad . \quad (14)$$

(*b*) **Basis Form:** *As in* (*a*) *above, i.e. no parallel middle in the half-body.*

Derived Form: *Required to introduce an amount of parallel middle equal to* δp *and change* ϕ *by* $\delta \phi$.

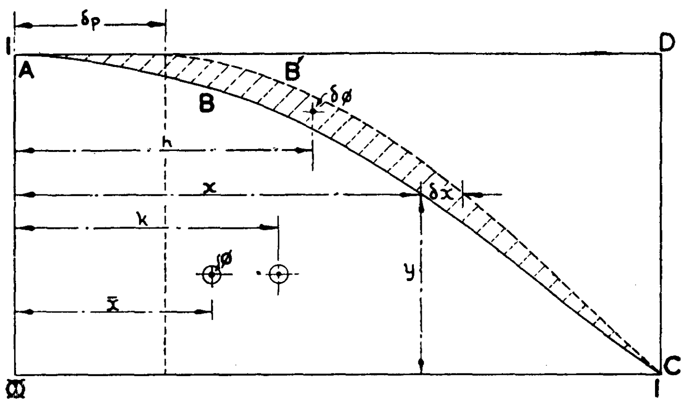

FIG. 4

The required relation between δx and x is:—

$$\delta x = (1-x)\left\{\delta p + \frac{[\delta\phi - \delta p(1-\phi)] \,.\, x}{\phi(1-2\bar{x})}\right\} \quad . \quad (15)$$

The first approximation to the lever h is:—

$$h = \left[\frac{1-\dfrac{\delta p}{\delta \phi}(1-\phi)}{1-2\bar{x}}\right](2\bar{x}-3k^2) + \frac{\delta p}{\delta \phi} \,.\, \phi(1-2\bar{x}) \quad . \quad . \quad . \quad (16)$$

The absolute limits of $\delta \phi$ are given by:—

$$\delta\phi = \delta p(1-\phi) \pm \phi(1-2\bar{x})(1-\delta p) \quad . \quad (17)$$

The corresponding practical limits are given by:—

$$\delta\phi = \delta p(1-\phi) \pm \tfrac{1}{2}\phi(1-2\bar{x})(1-\delta p) \quad . \quad (18)$$

(*c*) **Basis Form:** *Parallel middle in the half-body equal to p.*

Derived Form: *Required to change p by* δp *keeping* ϕ *constant.*

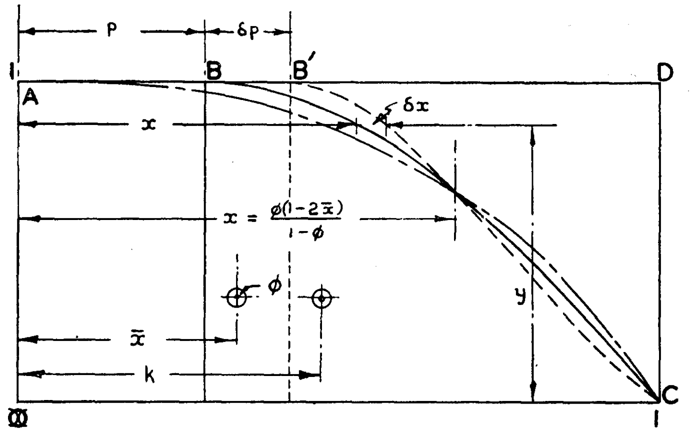

FIG. 5

The required relation between δx and x is:—

$$\delta x = \frac{\delta p(1-x)}{1-p}\left[1 - \frac{(1-\phi)(x-p)}{\phi(1-2\bar{x}) - p(1-\phi)}\right] \quad . \quad (19)$$

The derived curves will again cross the basis curve; the crossing point is given by:—

$$x = \frac{\phi(1-2\bar{x})}{(1-\phi)} \quad . \quad . \quad . \quad . \quad (20)$$

Again, as no area is added, the lever h is indeterminate, but the change in longitudinal movement of the centroid $\bar{x}$ is, to a first approximation, given by:—

$$\delta \bar{x} = \frac{-\delta p}{(1-p)}\left\{\frac{(1-\phi)[2\bar{x}-3k^2-p(1-2\bar{x})]}{\phi(1-2\bar{x})-p(1-\phi)} - (1-2\bar{x})\right\} \quad . \quad (21)$$

The absolute limits to which parallel middle body can be increased or reduced is given by:—

$$\delta p = \frac{1-p}{1 \pm \dfrac{(1-\phi)(1-p)}{\phi(1-2\bar{x})-p(1-\phi)}} \quad . \quad (22)$$

The corresponding practical limits will be given by:—

$$\delta p = \frac{1-p}{1 \pm \dfrac{2(1-\phi)(1-p)}{\phi(1-2\bar{x})-p(1-\phi)}} \quad \text{(approx.)} \quad . \quad (23)$$

An example of this variation is shown in Fig. 12, *vide* also remarks under section 9, "Summary of Illustrated Examples."

(*d*) **Basis Form:** *As in* (*c*) *above, i.e. parallel middle in the half-body equal to p.*

Derived Form: *Maintaining parallel middle body p constant and changing* ϕ *by* $\delta \phi$.

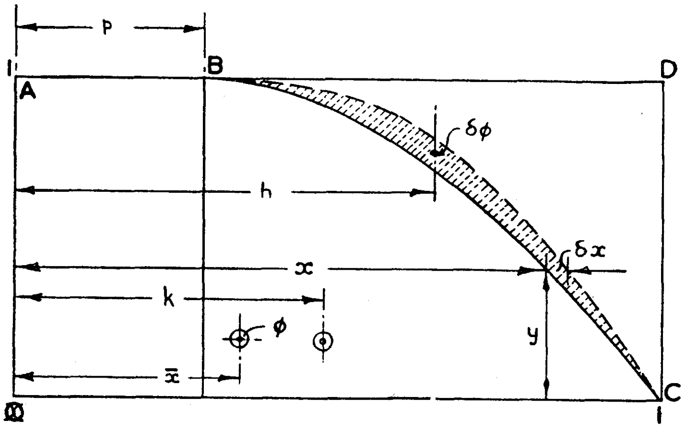

FIG. 6

This amounts to varying the fineness of the entrance (or run). The required relation between δx and x is:—

$$\delta x = \frac{\delta \phi (1 - x)(x - p)}{\phi (1 - 2\bar{x}) - p(1 - \phi)} \quad . \quad . \quad (24)$$

The corresponding first approximation to the lever h is:—

$$h = \frac{\phi [2\bar{x} - 3k^2 - p(1 - 2\bar{x})]}{\phi(1 - 2\bar{x}) - p(1 - \phi)} \quad . \quad . \quad (25)$$

The absolute limits of $\delta \phi$ in this case are given by:—

$$\delta \phi = \pm \frac{\phi (1 - 2\bar{x}) - p(1 - \phi)}{(1 - p)} \quad . \quad . \quad (26)$$

The corresponding practical limits are given by:—

$$\delta \phi = \pm \frac{\phi(1 - 2\bar{x}) - p(1 - \phi)}{2(1 - p)} \text{ (approx.)} \quad . \quad (27)$$

An example of this variation is shown in Fig. 13; *vide* also remarks under section 9, "Summary of Illustrated Examples."

5. General Case

Basis Form: *Any extent of parallel middle body.*

Derived Form: *Any required change in prismatic coefficient and extent of parallel middle body.*

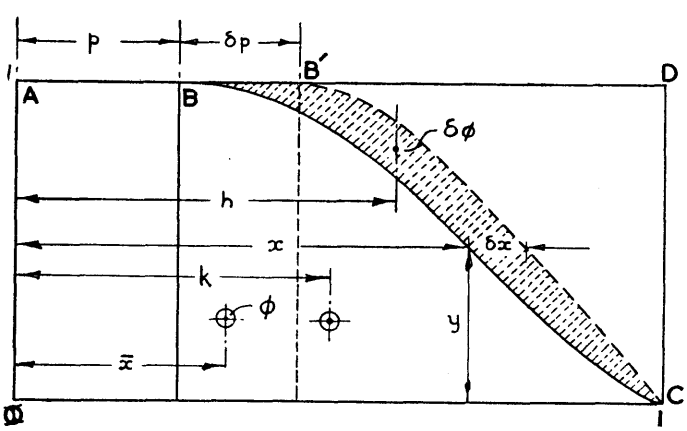

FIG. 7

The general relation between δx and x is:—

$$\delta x = (1 - x)\left\{\frac{\delta p}{1 - p} + \frac{(x - p)}{A}\left[\delta \phi - \delta p \frac{(1 - \phi)}{(1 - p)}\right]\right\} \quad . \quad . \quad . \quad (28)$$

The corresponding first approximation to the lever h is:—

$$h = \phi\left\{\frac{B}{\phi}\left[1 - \frac{\delta p (1 - \phi)}{\delta \phi (1 - p)}\right] + \frac{\delta p}{\delta \phi} \frac{(1 - 2\bar{x})}{(1 - p)}\right\} \quad . \quad (29)$$

The absolute limits of $\delta \phi$ for the general case are given by:—

$$\delta \phi = \frac{\delta p (1 - \phi) \pm A\left(1 - \frac{\delta p}{1 - p}\right)}{1 - p} \quad . \quad (30)$$

and the corresponding practical limits by:—

$$\delta \phi = \frac{\delta p (1 - \phi) \pm \frac{A}{2}\left(1 - \frac{\delta p}{1 - p}\right)}{1 - p} \text{ (approx.)} \quad . \quad (31)$$

where A and B are constants depending only on the geometrical properties of the basis form; these are as follows:—

$$A = \phi(1 - 2\bar{x}) - p(1 - \phi)$$

and $$B = \frac{\phi[2\bar{x} - 3k^2 - p(1 - 2\bar{x})]}{A}$$

For general use it may be found more convenient to use the comprehensive relations given in this section (*vide* also remarks under Case G of the worked examples in Appendix I).

All the special cases previously considered are included in these equations. For example the relations for δx and h for the "one minus prismatic" variation can be obtained by substituting the special condition that $\frac{\delta p}{1 - p}$ must equal $\frac{\delta \phi}{1 - \phi}$. The expressions for the other special cases considered can be obtained by making either δp, $\delta \phi$, p or combinations of these items equal to nought as appropriate.

6. Consideration of the Relative Fullness of the Fore and After Bodies as affecting the Position of the LCB

So far only the individual half-bodies have been dealt with. These will now be considered together in order to determine the effect of the variations discussed on the total prismatic coefficient and the position of the LCB.

Fig. 8 represents the complete sectional area curve, each half-body of which is one unit long and one unit maximum ordinate. The full line A B C represents the basis area curve and the dotted line the derived curve.

Let ϕ_t = the total prismatic coefficient of the basis ship,
$\delta \phi_t$ = the required change in total prismatic coefficient,
ϕ_f = the forebody prismatic coefficient of the basis ship,

ϕ_a = the afterbody prismatic coefficient of the basis ship,
$\delta\,\phi_f$ = the change in forebody prismatic coefficient,
$\delta\,\phi_a$ = the change in afterbody prismatic coefficient,
$\bar{z}$ = the distance of the LCB in the basis ship from midships expressed as a fraction of the half-length (positive forward of midships; negative aft),
$\delta\,\bar{z}$ = the required fractional shift of the LCB in the derived form (positive for movement forward; negative for movement aft).

(*a*) **General**

In general it can be shown that

$$\delta\,\phi_f = \frac{2\,[\delta\,\phi_t\,(h_a+\bar{z}) + \delta\,\bar{z}\,(\phi_t+\delta\,\phi_t)]}{(h_f+h_a)} \quad . \quad (32)$$

and
$$\delta\,\phi_a = \frac{2\,[\delta\,\phi_t\,(h_f-\bar{z}) - \delta\,\bar{z}\,(\phi_t+\delta\,\phi_t)]}{(h_f+h_a)} \quad . \quad (33)$$

calculation and remarks for the General Case G to which reference has already been made.

(*b*) **Position of LCB in derived form to be the same as in the basis ship**

In this case $\delta\,\bar{z}=0$ and the expressions for $\delta\,\phi_f$ and $\delta\,\phi_a$ reduce to the following:—

$$\delta\,\phi_f = \frac{2\,\delta\,\phi_t\,(h_a+\bar{z})}{h_f+h_a} \quad . \quad . \quad . \quad (34)$$

and
$$\delta\,\phi_a = \frac{2\,\delta\,\phi_t\,(h_f-\bar{z})}{h_f+h_a} \quad . \quad . \quad . \quad . \quad (35)$$

(*c*) **Required to change extent of parallel middle body keeping the prismatic coefficients of the fore and after bodies constant.**

It can be shown that

$$\delta\,\bar{z} = \frac{\phi_f\,.\,\delta\,\bar{x}_f - \phi_a\,.\,\delta\,\bar{x}_a}{2\,\phi_t} \quad . \quad . \quad . \quad (36)$$

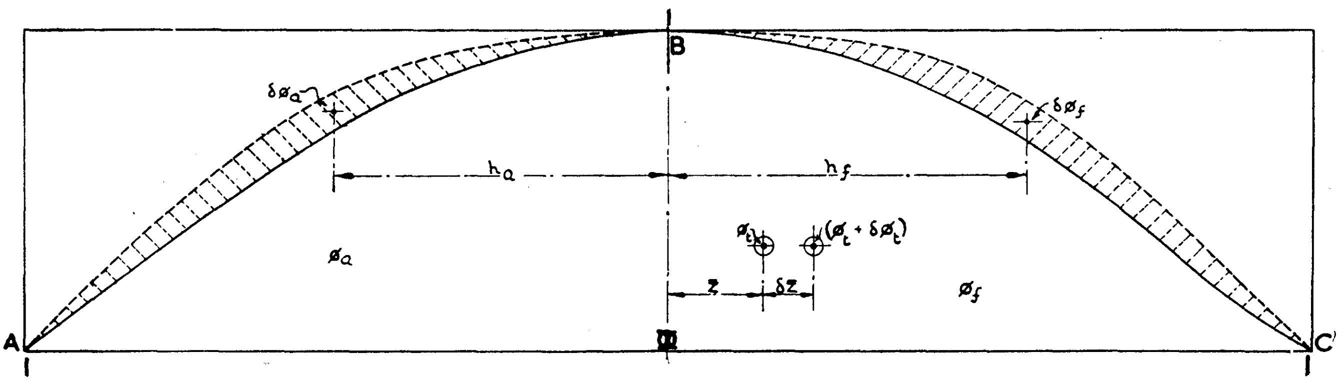

FIG. 8

The levers h_f and h_a for the fore and after bodies respectively can be calculated by the relations given in previous sections of the paper. Thus the required adjustments to the fore and after body prismatics to give any desired change in LCB position and total prismatic coefficient can be determined from the above equations for $\delta\,\phi_f$ and $\delta\,\phi_a$. From these the revised spacing of the transverse sections can be calculated using the appropriate expressions for $\delta\,x$.

It will be found that in all cases considered in previous sections of the paper the substitution in equations (32) and (33) of the first approximations to the levers h_f and h_a can be made directly except in the general case considered in section 5. In this case it will be seen from equation (29) that the relations for h_f and h_a will themselves involve the required changes in fineness $\delta\phi_f$ and $\delta\phi_a$. This difficulty has been overcome by substituting the expressions for h_f and h_a in equations (32) and (33) and solving for $\delta\,\phi_f$ and $\delta\,\phi_a$. This matter is dealt with in Case G of the "Worked Examples" given in Appendix I (*vide* equations (40) and (41) on p. 18 for these solutions for $\delta\,\phi_f$ and $\delta\,\phi_a$).

Convenient methods of evaluating equations (32) and (33) for various typical cases are given in Appendix I, and attention is drawn particularly to the form of

The movements of the centroids of the fore and after bodies $\delta\,\bar{x}_f$ and $\delta\,\bar{x}_a$ can be calculated in terms of the change of parallel middle body $\delta\,p$ from the relations given in section 4(*a*) or section 4(*c*) as appropriate.

If in making the change it is desired to keep the LCB in the same position then $\delta\,\bar{z}=0$, and therefore

$$\phi_f\,.\,\delta\,\bar{x}_f = \phi_a\,.\,\delta\,\bar{x}_a \quad . \quad . \quad . \quad . \quad (37)$$

(*d*) **Required to change the position of the LCB keeping the total prismatic coefficient constant**

In this case $\delta\,\phi_t=0$ and the expressions for $\delta\,\phi_f$ and $\delta\,\phi_a$ reduce to the following:—

$$\delta\,\phi_f = \frac{2\,\delta\,\bar{z}\,.\,\phi_t}{h_f+h_a} \quad . \quad . \quad . \quad . \quad (38)$$

and
$$\delta\,\phi_a = -\frac{2\,\delta\,\bar{z}\,.\,\phi_t}{h_f+h_a} \quad . \quad . \quad . \quad . \quad (39)$$

7. Method of Lifting the Sections for the Derived Form

As already explained, the derived form is obtained by re-spacing the transverse sections of the basis ship and the necessary shift of any particular section distance x

from midships is given by δx which can be readily calculated for any particular case. A convenient method of executing this transfer of sections will now be considered.

Fig. 9(*a*) represents the forebody area curve in a given case; the full line being the basis curve and the dotted line the derived curve. It is clear from the figure that to obtain the derived form the original section 9 must be displaced from A to B and the original section 7 must be displaced from A′ to B′ and so on; A B and A′ B′ being equal to δx at $x = 0{\cdot}8$ and $0{\cdot}4$ respectively. However, in order to be able to lift the new body sections directly from the basis half-breadth plan the

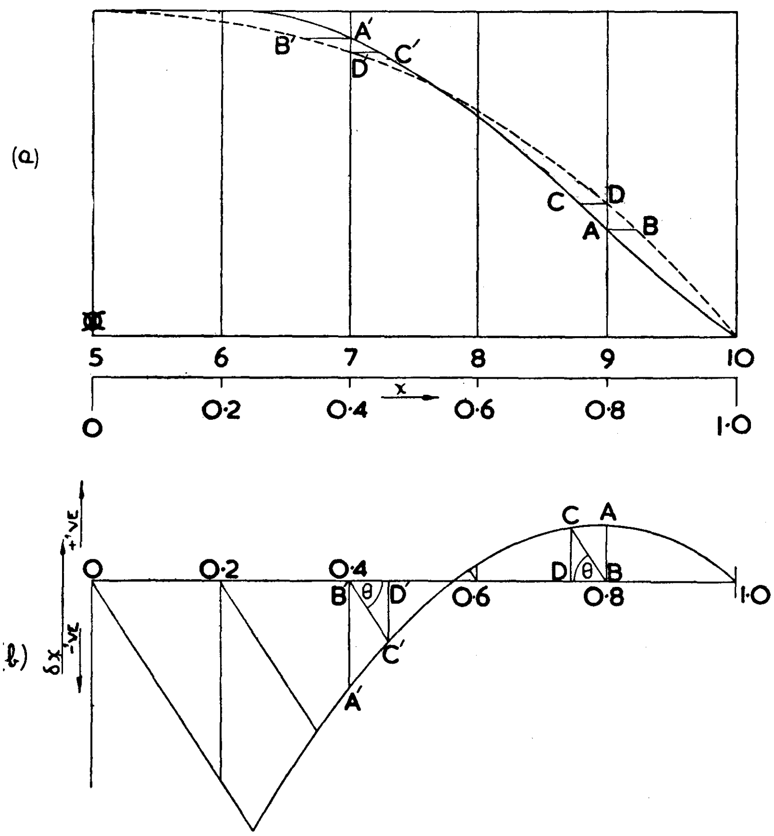

FIG. 9

shifts C D, C′ D′, etc., are required; i.e. the problem is to determine which intermediate sections of the basis ship will be the actual displacement sections in the derived form. This can best be done graphically as shown in Fig. 9(*b*). The shift δx is calculated for the various displacement stations 5, 6, 7, etc. (i.e. $x = 0$, $0{\cdot}2$, $0{\cdot}4$, etc.), and plotted on a contracted base of half-length as shown. If the vertical scale of δx is $1/\alpha_1$ full size and the horizontal scale of length is $1/\alpha_2$ full size then inclined lines are drawn as shown in Fig. 9(*b*) such that

$$\tan \theta = \frac{\alpha_2}{\alpha_1}$$

Verticals such as C D, C′ D′, etc., represent the required shifts and these can be accurately scaled from the diagram. The required water-line half-breadths can then be lifted directly from the lines plan for the basis ship. If the beam and draught of the new form differ from the basis ship, the half-breadths, as lifted, can be proportionally expanded or contracted by any convenient means and plotted on the corresponding water-lines in the body plan.

8. General

As already explained in section 3, when the fullness of the entrance or run is varied, this is effected without concentration of area towards either the shoulder or the ends; in other words the area curve is filled out or fined generally. This has been achieved by taking δx proportional to $x(1-x)$ which is a maximum when $x = \frac{1}{2}$. By taking a relation of the general form $x^n(1-x^m)$* the distribution of area over the entrance could be concentrated towards either end by suitable choice of m or n. The corresponding relations for δx and h are more complicated, however, and in any case the simple variation which has been used is the most suitable for general application. Of course when considering the general case for a half-body (*vide* section 5) the distribution of area along the length can be controlled to almost any extent by the amount of parallel middle body which is introduced or removed. In the particular case of the "one minus prismatic" variation the maximum shift of section δx is restricted to the shoulders of the area curve.

The development of body sections for a new design by the simple longitudinal shift of sections described in section 7 means that the midship section coefficient is necessarily the same in the basis and derived forms. However, the variations described in the paper can, of course, be used in general to vary the fullness, parallel middle body and LCB position of any area curve. If necessary the midship section coefficient could be adjusted without much trouble by fairing in the lower water-lines of the basis ship to the required shape of midship section and the body sections amended accordingly. The required variation could then be applied to this amended basis ship, due account being taken of the change in prismatic coefficient and other geometrical properties resulting from the new midship section coefficient.

9. Summary of Illustrated Examples

Several illustrated examples of the methods of form derivation described in the paper are given in Figs. 10 to 14, and for convenience these are summarized as follows:—

Figs. 10 and 11.—Varying the fullness of entrance area curves to the upper and lower limits.

Fig. 10 shows the forebody parent curve of the B.S.R.A. trawler series which, as there is no parallel middle body, is identical with the entrance curve. This has been filled out and fined to the absolute limits referred to in section 3 of the paper. The prismatic

* *Vide* author's reply to Dr. Hughes contribution to the discussion on p. 316.

coefficient of this curve is 0·637 and, as it is not associated with parallel middle body, the curve at midships falls away rather quickly. In this case it will be noted that derived curves of good shape are obtained when the variation in fineness is taken at least half way to the upper and lower limits. This represents a variation of $\pm$ 15 per cent on the prismatic coefficient of the entrance.

Fig. 11 shows the entrance curve of a merchant ship having a block coefficient of 0·70 which has been similarly varied. This curve is actually associated with parallel middle body and the fall of the curve in way of the shoulder is more gradual as a consequence. It would appear here that good derived curves are obtained when the variation in fineness is taken as far as three-quarters of the absolute limits. This represents a variation of about $\pm$ 22 per cent on the prismatic coefficient of the entrance which is 0·656.

As a general guide, therefore, it would appear to be well on the safe side to consider a practical limit of half the absolute, which should cover any variation in fullness likely to be required in a given case.

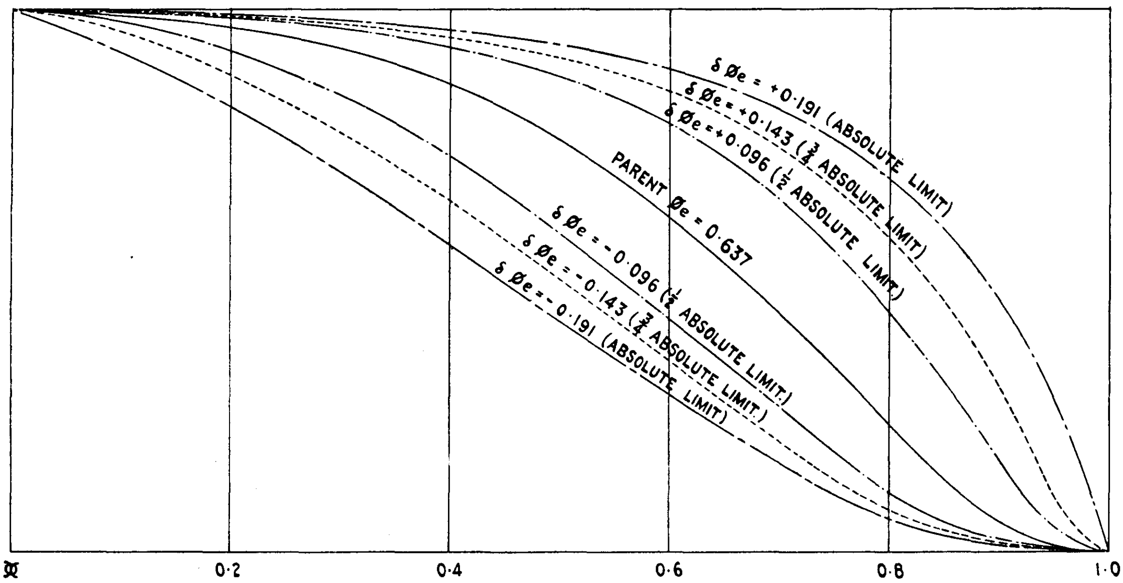

FIG. 10.—EXAMPLE OF VARYING THE FULLNESS OF ENTRANCE AREA CURVE TO THE UPPER AND LOWER LIMITS

Parent Curve.—The forebody area curve of the parent form of the B.S.R.A. Trawler Series. There is no parallel middle body in this form and the length of entrance is the same as the length of the forebody.

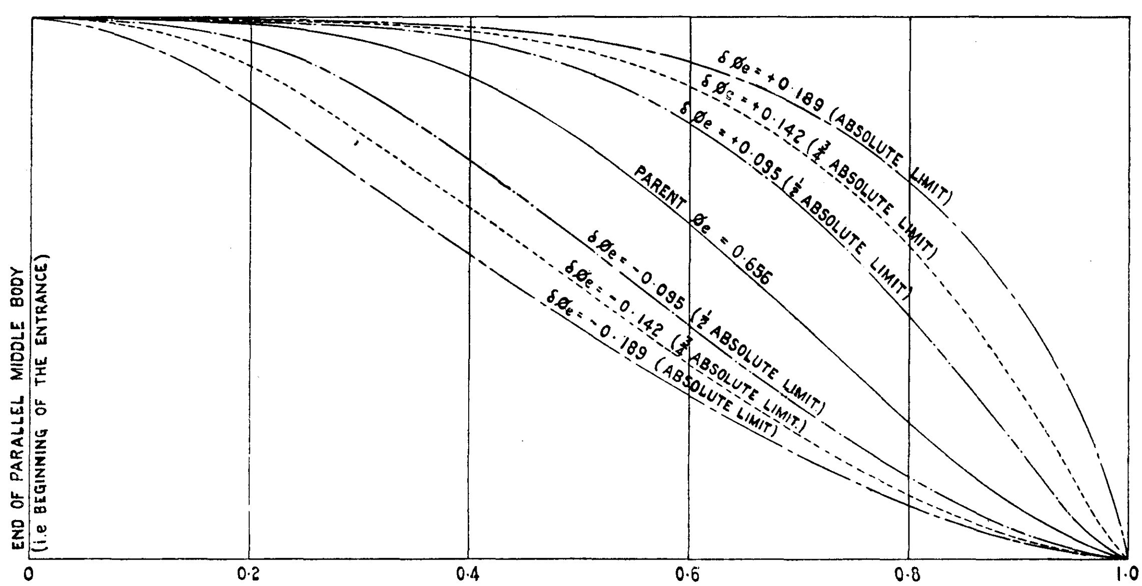

FIG. 11.—EXAMPLE OF VARYING THE FULLNESS OF ENTRANCE AREA CURVE TO THE UPPER AND LOWER LIMITS

Parent Curve.—The area curve of the entrance of a merchant ship having a block coefficient of 0·70. This is the basis ship used in the "Worked Examples" (Appendix I).

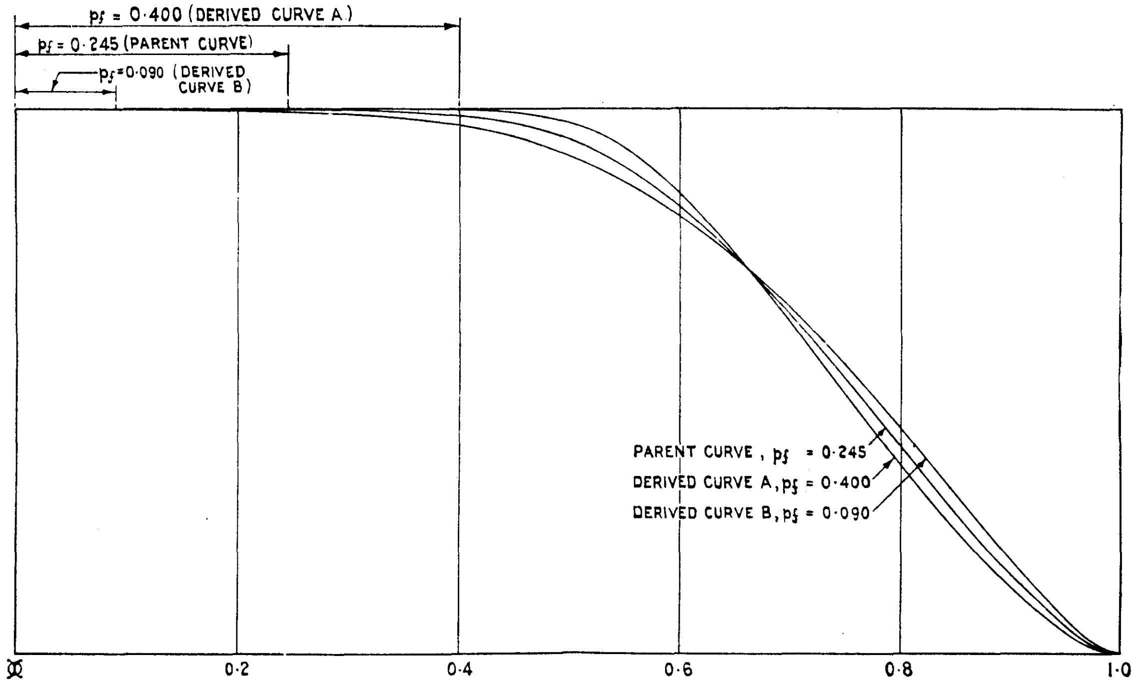

FIG. 12.—EXAMPLE OF FOREBODY VARIATIONS

Varying the extent of parallel middle body keeping the forebody prismatic coefficient constant.

Note.—In Figs. 12 and 13 the parent curve is the forebody area curve of the basis ship used in the "Worked Examples" (Appendix I). This is a merchant ship having a block coefficient of 0·70; the prismatic coefficient of the forebody $\phi_f = 0{\cdot}740$.

The extent of parallel middle in the forebody p_f is expressed as a fraction of the forebody length.

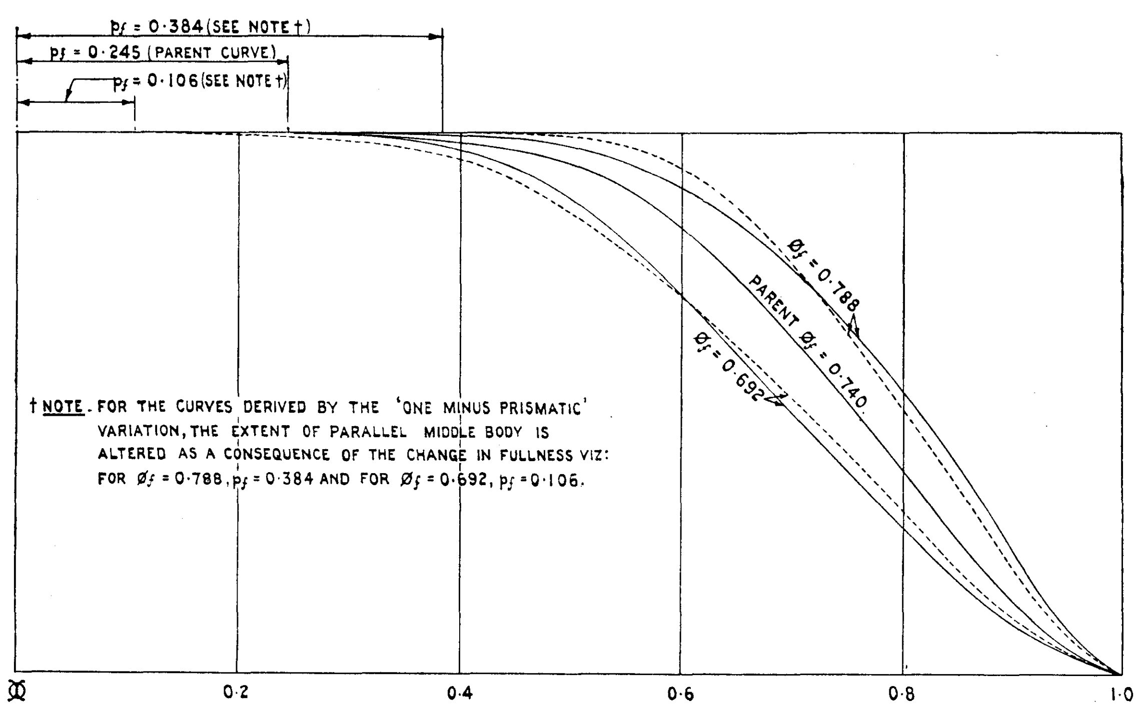

FIG. 13.—EXAMPLE OF FOREBODY VARIATIONS

Varying the prismatic coefficient of the forebody to 0·788 and 0·692.

Full Lines: Maintaining the original extent of parallel middle body using the more general methods described in the paper.
Dotted Lines: Using the usual "one minus prismatic" method of variation.

Figs. 12 and 13.—Forebody variations.

Fig. 12 shows the forebody area curve of the basis form referred to in connection with Fig. 11, in which the extent of parallel body has been varied keeping the prismatic coefficient of the forebody constant. The parent curve has 24·5 per cent parallel middle in the forebody (i.e. $p_f = 0{\cdot}245$) and this has been increased by 15·5 per cent in one case and reduced by the same amount in the other, the area under each curve being the same. This would appear to be a convenient and systematic method of "easing" or "hardening" the shoulder while keeping the displacement unaltered.

Fig. 13 shows the same forebody parent curve as in Fig. 12, and the prismatic coefficient of this has been varied to $\delta\,\phi_f \pm 0{\cdot}048$ by two different methods. In the case of the full lines, the extent of parallel middle body has been kept constant and in the case of the two dotted curves the variation has been carried out by the usual "one minus prismatic" method. It will be noted in the latter case that the extent of the parallel middle body is altered considerably.

Fig. 14.—Typical family of area curves having no parallel middle body.

A family of area curves is shown in which the fullness of the parent curve has been varied over the range $\delta\,\phi_t = \pm\,0{\cdot}05$ keeping the LCB position unchanged. The parent curve is the basic curve of areas of the B.S.R.A. trawler series, and it will be noted that this has no parallel middle body nor have the four curves derived from it. (*Note*: These are not the actual area curves for the Trawler Series.)

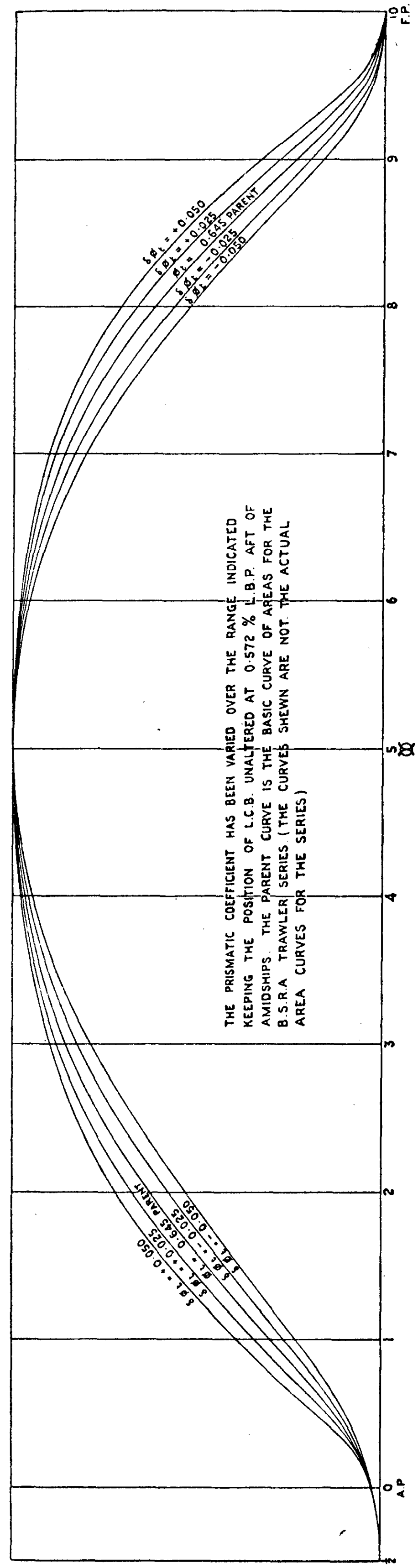

Fig. 14.—Typical family of area curves having no parallel middle body

10. Summary of Nomenclature

For a *Half-Body* Generally (either Forward or Aft)

In the Basis Ship:—

ϕ = the prismatic coefficient.

All expressed as a fraction of the length of the half-body:

- p = the extent of parallel middle body.
- $\bar{x}$ = the lever of the first moment (i.e. the centroid of the half-body) about midships.
- k = the lever of the second moment (i.e. the radius of gyration) about midships.
- r = the lever of the third moment about midships.
- x = the distance of any transverse section from midships.

y = the area of the transverse section at x expressed as a fraction of the maximum area.

$$A = \phi\,(1 - 2\,.\,\bar{x}) - p\,(1 - \phi)$$

$$B = \frac{\phi\,[2\,.\,\bar{x} - 3\,.\,k^2 - p(1 - 2\,.\,\bar{x})]}{A}$$

$$C = \frac{B\,(1 - \phi) - \phi\,(1 - 2\,.\,\bar{x})}{1 - p}$$

Vide Appendix I ("Worked Examples") for the applications of these form constants A, B, and C.

In the Derived Form:—

Changes in ϕ, p and $\bar{x}$ are denoted by $\delta\phi$, δp and $\delta\bar{x}$. These changes can be either positive or negative and are not necessarily "small" quantities in the mathematical sense (except for $\delta\phi$ which is assumed to be relatively small when using the first approximation to the lever h).

δx = the necessary longitudinal shift of the section at x to produce the required changes in ϕ, p and LCB position. (If δx is positive the shift is away from midships; if negative the shift is towards midships.)

h = the distance from midships of the centroid of the "sliver" of area (represented by $\delta\phi$) which is added to a half-body area curve of the basis ship. This is also expressed as a fraction of the length of the half-body.

Note.—Subscripts f and a are added to the above symbols to denote the fore and after bodies respectively.

For the Ship Form as a Whole

In the Basis Ship:—

ϕ_t = the total prismatic coefficient $= \frac{1}{2}(\phi_f + \phi_a)$.

$\bar{z}$ = the distance of the LCB from midships expressed as a fraction of the *length of a half-body* (positive forward of midships; negative aft).

In the Derived Form:—

$\delta\phi_t$ = the required change in total prismatic coefficient $= \frac{1}{2}(\delta\phi_f + \delta\phi_a)$.

$\delta\bar{z}$ = the required change in LCB position (positive for movement forward; negative for movement aft).

Relations for Prismatic Coefficients of Entrance and Run

ϕ_e = the prismatic coefficient of the entrance $= \dfrac{\phi_f - p_f}{1 - p_f}$

ϕ_r = the prismatic coefficient of the run $= \dfrac{\phi_a - p_a}{1 - p_a}$

The lengths of the entrance and run expressed as a fraction of the length of a half-body are given by $(1 - p_f)$ and $(1 - p_a)$ respectively.

Appendix I
WORKED EXAMPLES
Summary of Examples

"One Minus Prismatic" Variations

Case A.—Varying the fullness keeping the LCB position unchanged.

Case B.—Varying the fullness and LCB position.

More General Methods of Form Variation

Case C.—Varying the fullness keeping the LCB position and extent of parallel middle body unchanged.

Case D.—Varying the fullness and LCB position keeping the extent of parallel middle body unchanged.

Case E.—Varying the position of the LCB keeping the fullness and extent of parallel middle body unchanged.

Case F.—Varying the extent of parallel middle body keeping the fullness and LCB position unchanged.

Case G.—A general case in which the fullness, LCB position, and extent of parallel middle in both fore and after bodies are all varied.

TABLE I

1	2	3	4	5	6	7
Station	Fractional area	S.M.	Function of volume (2 × 3)	Fractional lever	Functions of moments: First (4 × 5)	Functions of moments: Second (6 × 5)
A.P. 0	0·015	$\frac{1}{4}$	0·004	1	0·0038	0·0038
$\frac{1}{4}$	0·069	1	0·069	0·95	0·0656	0·0623
$\frac{1}{2}$	0·151	$\frac{1}{2}$	0·076	0·90	0·0684	0·0616
$\frac{3}{4}$	0·236	1	0·236	0·85	0·2006	0·1705
1	0·328	$\frac{3}{4}$	0·246	0·80	0·1967	0·1574
$1\frac{1}{2}$	0·510	2	1·020	0·70	0·7140	0·4998
2	0·688	1	0·688	0·60	0·4128	0·2477
$2\frac{1}{2}$	0·826	2	1·652	0·50	0·8260	0·4130
3	0·921	$1\frac{1}{2}$	1·382	0·40	0·5528	0·2211
4	0·999	4	3·996	0·20	0·7992	0·1598
5	1·000	2	1·000 / 1·000	0	3·8399	1·9970
6	1·000	4	4·000	0·20	0·8000	0·1600
7	0·988	$1\frac{1}{2}$	1·482	0·40	0·5928	0·2371
$7\frac{1}{2}$	0·944	2	1·888	0·50	0·9440	0·4720
8	0·819	1	0·819	0·60	0·4914	0·2948
$8\frac{1}{2}$	0·623	2	1·246	0·70	0·8722	0·6105
9	0·381	$\frac{3}{4}$	0·286	0·80	0·2288	0·1830
$9\frac{1}{4}$	0·258	1	0·258	0·85	0·2193	0·1864
$9\frac{1}{2}$	0·141	$\frac{1}{2}$	0·072	0·90	0·0648	0·0583
$9\frac{3}{4}$	0·051	1	0·051	0·95	0·0485	0·0461
F.P. 10	—	$\frac{1}{4}$	—	1	—	—
	Afterbody ..		10·369		4·2618	2·2482
	Forebody ..		11·102			
	Total ..		21·471			

Sum of Simpson's Multipliers (Column 3) = 30.

Particulars of Basis Ship

A typical merchant ship form has been used as the basis throughout these examples, the principal particulars of which are as follows:—

400 ft. length B.P. $\times$ 55 ft. beam $\times$ 26 ft. draught $\times$ 11,460 tons displacement.

Block coefficient $= \dfrac{11{,}460 \times 35}{400 \times 55 \times 26} = 0{\cdot}7015$.

The mid-section coefficient is 0·9802, therefore the total prismatic coefficient $\phi_t = \dfrac{0{\cdot}7015}{0{\cdot}9802} = 0{\cdot}7157$.

The parallel middle in the forebody is 49 ft., therefore $p_f = 49/200 = 0{\cdot}245$.

The parallel middle in the afterbody is 8 ft., therefore $p_a = 8/200 = 0{\cdot}040$.

The ordinates of the sectional area curve expressed as a fraction of the maximum ordinate are as shown in column 2 of Table I.

Determination of the Necessary Geometrical Particulars of the Form

Referring to the form of calculation shown in Table I we have:—

Total prismatic coefficient $= \dfrac{21{\cdot}471}{30} = 0{\cdot}7157 = \phi_t$.

LCB forward of midships as a fraction of the half-length $= \dfrac{4{\cdot}2618 - 3{\cdot}8399}{21{\cdot}471} = +\,0{\cdot}0197 = \bar{z}$ (i.e. 3·94 ft. or 0·985 per cent LBP; the + sign indicates *forward* of midships).

Calculation of Forebody Particulars

Forebody prismatic coefficient $= 11{\cdot}102/15 = 0{\cdot}7401 = \phi_f$.

Lever of 1st moment about midships $= 4{\cdot}2618/11{\cdot}102 = 0{\cdot}3839 = \bar{x}_f$.

Lever (squared) of 2nd moment about midships $= 2{\cdot}2482/11{\cdot}102 = 0{\cdot}2025 = k_f^2$.

The corresponding items for the afterbody are found in a similar manner. The necessary geometrical particulars for both bodies are summarized in the following Table II:—

TABLE II

Items	Afterbody	Forebody
Prismatic coefficient ..	$\phi_a = 0{\cdot}6912$	$\phi_f = 0{\cdot}7401$
Lever of 1st Moment	$\bar{x}_a = 0{\cdot}3703$	$\bar{x}_f = 0{\cdot}3839$
Lever of 2nd Moment (squared)	$k_a^2 = 0{\cdot}1926$	$k_f^2 = 0{\cdot}2025$
Parallel middle in each body	$p_a = 0{\cdot}040$	$p_f = 0{\cdot}245$
Total prismatic coefficient ϕ_t=0·7157; LCB $\bar{z}$ =+0·0197		
Block coefficient=0·7015; Mid-section coefficient=0·9802		

Examples using the "One Minus Prismatic" Variation

Case A—Suppose it is required to increase the block coefficient to 0·725 using the usual "one minus prismatic" variation and that the LCB is to be kept in the same position.

(*Note*.—In this example general reference is made to section 2 of the paper.)

The required increase in block coefficient

$$= 0{\cdot}7250 - 0{\cdot}7015 = 0{\cdot}0235$$

Therefore the required increase in total prismatic coefficient $= \dfrac{0{\cdot}0235}{0{\cdot}9802} = +\,0{\cdot}0239 = \delta\,\phi_t$.

It is first necessary to calculate the levers h_f and h_a of the added "slivers" of area in the fore and after bodies so that the changes in the prismatic coefficients of the bodies ($\delta\,\phi_f$ and $\delta\,\phi_a$) can be determined. For this particular case the lever h is given by equation (4), viz.:—

$$h = \frac{\phi\,(1 - 2\,\bar{x})}{1 - \phi} \text{ (first approx.)}$$

$$\text{Therefore } h_f = \frac{0{\cdot}7401\,(1 - 2 \times 0{\cdot}3839)}{1 - 0{\cdot}7401} = 0{\cdot}662$$

$$\text{and} \qquad h_a = \frac{0{\cdot}6912\,(1 - 2 \times 0{\cdot}3703)}{1 - 0{\cdot}6912} = 0{\cdot}581$$

Since the position of the LCB is to remain unaltered then $\delta\,\bar{z} = 0$, and using equations (34) and (35), section 6, we have

$$\delta\,\phi_f = \frac{2\,\delta\,\phi_t\,(h_a + \bar{z})}{h_f + h_a} = \frac{2 \times 0{\cdot}0239\,(0{\cdot}581 + 0{\cdot}0197)}{0{\cdot}662 + 0{\cdot}581} = +\,0{\cdot}0231$$

and

$$\delta\,\phi_a = \frac{2\,\delta\,\phi_t\,(h_f - \bar{z})}{h_f + h_a} = \frac{2 \times 0{\cdot}0239\,(0{\cdot}662 - 0{\cdot}0197)}{0{\cdot}662 + 0{\cdot}581} = +\,0{\cdot}0247$$

The shift of sections in each body is given by equation (1),

$$\text{i.e.} \qquad \delta\,x = \frac{\delta\,\phi}{1 - \phi}\,(1 - x)$$

where x is the distance from midships of any section in the body expressed as a fraction of the half-length; ($\delta\,x$ is similarly a fraction of the half-length).

$$\text{Hence} \quad \delta\,x_f = \frac{+\,0{\cdot}0231}{1 - 0{\cdot}7401}\,(1 - x) = +\,0{\cdot}0889\,(1 - x)$$

$$\text{and} \quad \delta\,x_a = \frac{+\,0{\cdot}0247}{1 - 0{\cdot}6912}\,(1 - x) = +\,0{\cdot}0800\,(1 - x)$$

To obtain the actual shifts in feet these must be multiplied by the length of half-body, viz. 200 ft.

Thus shift of sections in forebody $= +\,17{\cdot}78\,(1 - x)$ ft. and shift of sections in afterbody $= +\,16{\cdot}00\,(1 - x)$ ft.

The + sign indicates movement *away* from midships.

These shifts have been plotted on a contracted base of length in Fig. 15 and, as described in section 7, the intermediate sections of the basis ship which will be the actual displacement sections of the derived form can be easily and accurately determined. For example, referring to Fig. 15:—

(i) Displacement section 3 in the derived form will be the same as the section 10·5 ft. *forward* of the original section 3 in the parent form.

(ii) Similarly the new section 8 will be the same as that 7·9 ft. *aft* of the original section 8 in the parent form, and so on for other sections.

It will be seen that this enables the displacement sections for the new form to be lifted directly from the half-breadth plan of the parent form without drawing out either the new sectional area or water-line curves.

Referring again to Fig. 15, it will be seen that the parallel middle body has been increased in making the variation. For each body the increase in parallel middle is given by equation (2), section 2, i.e. $\delta p = \delta \phi \dfrac{(1-p)}{(1-\phi)}$

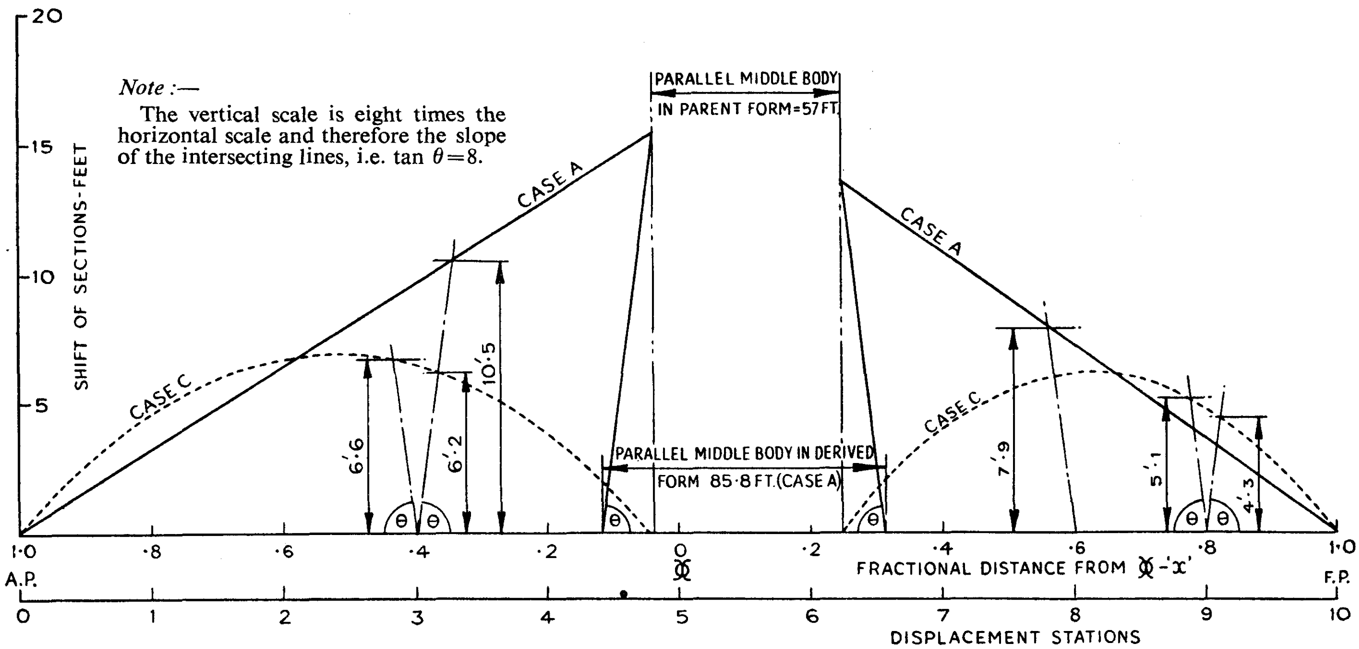

FIG. 15.—SHIFT OF SECTIONS FOR CASES A AND C

Case A.—Block coefficient increased by 0·025; LCB position unchanged (one minus prismatic variation).
Case C.—Block coefficient increased by 0·025; LCB position unchanged; extent of parallel middle body in parent form maintained.
(*Note.*—For details see Appendix I, "Worked Examples.")

Therefore $\delta p_f = \dfrac{0{\cdot}0231 \times (1 - 0{\cdot}245)}{(1 - 0{\cdot}7401)} = 0{\cdot}0671$

(i.e. $200 \times 0{\cdot}0671 = 13{\cdot}42$ ft.)

and $\delta p_a = \dfrac{0{\cdot}0247 \times (1 - 0{\cdot}040)}{(1 - 0{\cdot}6912)} = 0{\cdot}0768$

(i.e. $200 \times 0{\cdot}0768 = 15{\cdot}36$ ft.)

In making the variation, therefore, the parallel middle body has been increased from 57 ft. to 85·78 ft. As already explained in the paper this is the essential feature of this particular variation, i.e. the fullness of the form has been increased by simply inserting additional parallel middle body and contracting the ends. If the block coefficient was *reduced* by 0·025 by this method the parallel middle in the afterbody would become "negative." This could be overcome, however, by using the other more general methods of form variation described in the paper and illustrated by succeeding examples C to G.

Case B.—As in Case A above except that in making the variation it is required to move the LCB 2 ft. aft of its original position.

(*Note.*—General reference is again made to section 2 of the paper.)

$\delta \bar{z} = -\dfrac{2}{200} = -0{\cdot}01$ (negative sign indicates movement aft). As before $\delta \phi_t = 0{\cdot}0239$; $h_f = 0{\cdot}662$ and $h_a = 0{\cdot}581$ (first approx.). The required changes in the fore and after body prismatic coefficients are now given by equations (32) and (33), section 6(*a*), viz.:—

$$\delta \phi_f = \frac{2\,[\delta \phi_t (h_a + \bar{z}) + \delta \bar{z}\,(\phi_t + \delta \phi_t)]}{h_f + h_a}$$

$$= \frac{2\,[0{\cdot}0239\,(0{\cdot}581 + 0{\cdot}0197) - 0{\cdot}01\,(0{\cdot}7396)]}{0{\cdot}662 + 0{\cdot}581}$$

i.e. $\delta \phi_f = +\,0{\cdot}0112;$

and
$$\delta \phi_a = \frac{2\,[\delta \phi_t (h_f - \bar{z}) - \delta \bar{z}\,(\phi_t + \delta \phi_t)]}{h_f + h_a}$$

$$= \frac{2\,[0{\cdot}0239\,(0{\cdot}662 - 0{\cdot}0197) + 0{\cdot}01\,(0{\cdot}7396)]}{0{\cdot}662 + 0{\cdot}581}$$

i.e. $\delta \phi_a = +\,0{\cdot}0366$

The effect of using the exact levers h will now be shown. From section 2 of the paper it will be seen that the exact value is given by equation (3) as follows:—

$$h = \frac{\phi\,(1 - 2\bar{x})}{(1-\phi)} + \frac{\delta \phi}{2\,(1-\phi)^2}\,[1 - 2\,\phi\,(1 - \bar{x})]$$

When evaluated for the fore and after bodies we have:—

$h_f = 0{\cdot}662 + 0{\cdot}658\,\delta\,\phi_f$ and $h_a = 0{\cdot}581 + 0{\cdot}682\,\delta\,\phi_a$

Using the values of $\delta\,\phi_f$ and $\delta\,\phi_a$ determined above, the second approximations to the levers are

$$h_f = 0{\cdot}662 + (0{\cdot}658 \times 0{\cdot}0112) = 0{\cdot}669$$

and $$h_a = 0{\cdot}581 + (0{\cdot}682 \times 0{\cdot}0366) = 0{\cdot}606$$

Using these amended levers the second approximations to $\delta\,\phi_f$ and $\delta\,\phi_a$ are as follows:—

$$\delta\,\phi_f = \frac{2[0{\cdot}0239(0{\cdot}6060+0{\cdot}0197)-0{\cdot}01(0{\cdot}7396)]}{0{\cdot}669 + 0{\cdot}606} = +0{\cdot}0118$$

$$\delta\,\phi_a = \frac{2[0{\cdot}0239(0{\cdot}669-0{\cdot}0197)+0{\cdot}01(0{\cdot}7396)]}{0{\cdot}669 + 0{\cdot}606} = +0{\cdot}0360$$

It will be seen that only the fourth decimal place is affected. Any degree of accuracy can be obtained by carrying out similar successive approximations. Using the first approximation the LCB is moved aft 2·1 ft. instead of 2 ft.; using the second approximation the shift is actually 2 ft. aft. The change of fineness is exact in both cases.

The corresponding shifts of sections are given by equation (1) as follows:—

Forebody:

$$\frac{+\,0{\cdot}0118}{1 - 0{\cdot}7401}(1 - x) \times 200 \text{ ft.} = 9{\cdot}1\,(1 - x) \text{ ft.}$$

Afterbody:

$$\frac{+\,0{\cdot}0360}{1 - 0{\cdot}6912}(1 - x) \times 200 \text{ ft.} = 23{\cdot}3\,(1 - x) \text{ ft.}$$

The corresponding increases in parallel middle body calculated from equation (2) are:—

Forebody:

$$\delta\,p_f = \frac{+\,0{\cdot}0118\,(1 - 0{\cdot}245)}{(1 - 0{\cdot}7401)} = 0{\cdot}0343$$

(i.e. $200 \times 0{\cdot}0343 = 6{\cdot}86$ ft.)

Afterbody:

$$\delta\,p_a = \frac{+\,0{\cdot}0360\,(1 - 0{\cdot}04)}{(1 - 0{\cdot}6912)} = 0{\cdot}112$$

(i.e. $200 \times 0{\cdot}112 = 22{\cdot}4$ ft.)

For convenience the necessary relations for applying the "one minus prismatic" variation are summarized below:—

Summary of Relations for the "One Minus Prismatic" Variation

$$\delta\,\phi_f = \frac{2\,[\delta\,\phi_t\,(h_a + \bar{z}) + \delta\,\bar{z}\,(\phi_t + \delta\,\phi_t)]}{(h_f + h_a)} \quad . \quad (32)$$

$$\delta\,\phi_a = \frac{2\,[\delta\,\phi_t\,(h_f - z) - \delta\,\bar{z}\,(\phi_t + \delta\,\phi_t)]}{(h_f + h_a)} \quad . \quad (33)$$

(The items in the following expressions refer to the fore or after bodies as appropriate)

where the first approximation to the lever $h = \dfrac{\phi\,(1 - 2\,\bar{x})}{(1 - \phi)}$ . . . . . (4)

the exact lever h

$$= \frac{\phi\,(1 - 2\,\bar{x})}{(1 - \phi)} + \frac{\delta\,\phi}{2\,(1 - \phi)^2}[1 - 2\,\phi\,(1 - \bar{x})] \quad . \quad (3)$$

The shift of sections in either half-body:—

$$\delta\,x = \frac{\delta\,\phi}{1 - \phi}(1 - x) \quad . \quad . \quad . \quad . \quad (1)$$

The *consequent* change in extent of parallel middle in each body:—

$$\delta\,p = \frac{\delta\,\phi}{1 - \phi}(1 - p)\,. \quad . \quad . \quad . \quad . \quad (2)$$

Examples using the More General Methods of Form Variation in which the Extent of Parallel Middle Body is Controlled

In the examples which follow there are certain recurrent expressions involving only the geometrical characteristics of the parent form and the work is facilitated considerably by evaluating these beforehand. They are:—

$$\text{A} = \phi\,(1 - 2\,\bar{x}) - p\,(1 - \phi)$$

$$\text{B} = \frac{\phi\,[2\,\bar{x} - 3\,k^2 - p\,(1 - 2\,\bar{x})]}{\text{A}}$$

and $$\text{C} = \frac{\text{B}\,(1 - \phi) - \phi\,(1 - 2\,\bar{x})}{1 - p}$$

where $\phi, \bar{x}, k$, etc., refer to the fore or after body as appropriate.

Evaluating these constants A, B, C, from the particulars given in Table II we have:—

Afterbody	*Forebody*
$\text{A}_a = 0{\cdot}1670$	$\text{A}_f = 0{\cdot}1082$
$\text{B}_a = 0{\cdot}6310$	$\text{B}_f = 0{\cdot}7075$
$\text{C}_a = 0{\cdot}0162$	$\text{C}_f = 0{\cdot}0160$

Case C.—Block coefficient again to be increased by 0·025; LCB and extent of parallel middle body to remain unaltered.

(*Note.*—In this example general reference is made to section 4(*d*) of the paper.)

It will be seen from equation (25) that in this case the lever h is given by the constant B above.

Therefore $h_f = 0{\cdot}7075$ and $h_a = 0{\cdot}6310$.

From equations (34) and (35), section 6(*b*),

$$\delta\,\phi_f = \frac{2\,\delta\,\phi_t\,(h_a + \bar{z})}{h_f + h_a} = \frac{2 \times 0{\cdot}0239\,(0{\cdot}6310 + 0{\cdot}0197)}{0{\cdot}7075 + 0{\cdot}6310}$$

$$= 0{\cdot}0232$$

and $$\delta\,\phi_a = \frac{2\,\delta\,\phi_t\,(h_f - \bar{z})}{h_f + h_a} = \frac{2 \times 0{\cdot}0239\,(0{\cdot}7075 - 0{\cdot}0197)}{0{\cdot}7075 + 0{\cdot}6310}$$

$$= 0{\cdot}0246$$

For this case the shift of sections is given by equation (24), viz.:—

$$\delta\,x = \frac{\delta\,\phi\,(1 - x)\,(x - p)}{\text{A}}$$

The shifts are therefore:—

Forebody:

$$\frac{0{\cdot}0232\,(1 - x)\,(x - 0{\cdot}245) \times 200 \text{ ft.}}{0{\cdot}1082}$$

$$= 42{\cdot}9\,(1 - x)\,(x - 0{\cdot}245) \text{ ft.}$$

Afterbody:

$$\frac{0 \cdot 0246\,(1 - x)\,(x - 0 \cdot 04) \times 200 \text{ ft.}}{0 \cdot 1670}$$

$$= 29 \cdot 5\,(1 - x)\,(x - 0 \cdot 04) \text{ ft.}$$

These shifts have again been plotted on the same contracted scale of length in Fig. 15 and it will be seen that the parallel middle body has been unaltered. The positions of the new displacement sections can again be read off the curves. For example:—

(i) The new section 3 will be the same as that 6·2 ft. *forward* of the original section 3 in the parent form.

(ii) The new section 9 will be the same as that 5·1 ft. *aft* of the original section 9, and so on.

If it is required to *reduce* the block coefficient by 0·025 under the same conditions the curves of shift will be exactly the same as plotted except that the shift will be *towards* midships and the inclined intersecting lines will have the reverse slope as drawn at the sections 3 and 9. That is:—

(i) The new section 3 will be the same as that 6·6 ft. *aft* of the original section 3.

(ii) The new section 9 will be the same as that 4·3 ft. *forward* of the original section 9, and so on.

Check on the Limits of the Fineness Variation

For this case the practical limits of $\delta\phi$ for either body are given by equation (27), i.e. $\delta\phi = \pm\dfrac{A}{2\,(1 - p)}$.

These limits are therefore:—

Forebody:

$$\delta\phi_f = \pm\frac{0 \cdot 1082}{2\,(1 - 0 \cdot 245)} = \pm\,0 \cdot 0717$$

Afterbody:

$$\delta\phi_a = \pm\frac{0 \cdot 1670}{2\,(1 - 0 \cdot 04)} = \pm\,0 \cdot 0870$$

It will be seen that the variation is well within these limits.

Case D.—The same as Case C above except that in making the variation the LCB is to be moved 2 ft. aft of its original position.

(*Note*.—General reference is again made to section 4(*d*).)

$$\delta\bar{z} = \frac{-2}{200} = -\,0 \cdot 01; \text{ as before } \delta\phi_t = +\,0 \cdot 0239$$

h_f and h_a will be the same as in Case C, viz. 0·7075 and 0·6310 respectively.

From equations (32) and (33), section 6(*a*):—

$$\delta\phi_f = \frac{2\,[\delta\phi_t\,(h_a + \bar{z}) + \delta\bar{z}\,(\phi_t + \delta\phi_t)]}{h_f + h_a}$$

$$= \frac{2\,[0 \cdot 0239\,(0 \cdot 631 + 0 \cdot 0197) - 0 \cdot 01\,(0 \cdot 7396)]}{0 \cdot 7075 + 0 \cdot 6310} = +\,0 \cdot 0122$$

and
$$\delta\phi_c = \frac{2\,[\delta\phi_t\,(h_f - \bar{z}) - \delta\bar{z}\,(\phi_t + \delta\phi_t)]}{h + h_a}$$

$$= \frac{2\,[0 \cdot 0239\,(0 \cdot 7075 - 0 \cdot 0197) + 0 \cdot 01\,(0 \cdot 7396)]}{0 \cdot 7075 + 0 \cdot 6310} = +\,0 \cdot 0356$$

The shifts corresponding to these changes in fineness of the fore and after bodies are calculated as in Case C. In feet they are:—

Forebody: $+\,22 \cdot 6\,(1 - x)\,(x - 0 \cdot 245)$ ft.
Afterbody: $+\,42 \cdot 7\,(1 - x)\,(x - 0 \cdot 04)$ ft.

It will be noted that the changes in fineness are still well within the limits calculated in Case C above.

Case E.—Suppose it is required to move the LCB only, i.e. fineness and extent of parallel middle body to remain unaltered.

(*Note*.—General reference is again made to section 4(*d*).)

Let the required LCB change be 4 ft. forward of the original position, i.e. $\delta\bar{z} = \dfrac{+4}{200} = +\,0 \cdot 02$ (+ sign indicates movement forward); h_f and h_a will be the same as in the last two cases, viz. 0·7075 and 0·6310 respectively.

For this case we have from equations (38) and (39), section 6(*d*):—

$$\delta\phi_f = \frac{2\,\delta\bar{z}\,.\,\phi_t}{h_f + h_a} = \frac{2 \times 0 \cdot 02 \times 0 \cdot 7157}{0 \cdot 7075 + 0 \cdot 6310} = +\,0 \cdot 0214$$

$$\delta\phi_a = \frac{-\,2\,\delta\bar{z}\,.\,\phi_t}{h_f + h_a} = -\,0 \cdot 0214$$

These changes in fineness are again well within the limits calculated in Case C above.

As before the shift of sections in each body is given by equation (24), i.e. $\delta x = \dfrac{\delta\phi\,(1 - x)\,(x - p)}{A} \times 200$ ft.

The required shifts are therefore:—

Forebody:

$$\frac{+\,0 \cdot 0214\,(1 - x)\,(x - 0 \cdot 245) \times 200}{0 \cdot 1082}$$

$$= +\,39 \cdot 6\,(1 - x)\,(x - 0 \cdot 245) \text{ ft.}$$

Afterbody:

$$\frac{-\,0 \cdot 0214\,(1 - x)\,(x - 0 \cdot 04) \times 200}{0 \cdot 1670}$$

$$= -\,25 \cdot 6\,(1 - x)\,(x - 0 \cdot 04) \text{ ft.}$$

This simple LCB variation could also be carried out by the well-known method of "swinging" the area curve (*vide* Appendix III). Here again, however, as in the "one minus prismatic" variation, there would be no control over the position of the parallel middle body. This would be pushed forward bodily; the extent in the forebody would be increased by a definite amount and the extent in the afterbody would be reduced by the same amount.

Case F.—Suppose it is required to increase the extent of parallel middle body in the parent form from 57 ft. to 100 ft.; fineness in both bodies and LCB position to remain unaltered.

(*Note*.—In this example general reference is made to section 4(*c*) of the paper.)

Let increase in parallel middle in forebody = δp_f.
Let increase in parallel middle in afterbody = δp_a.

Then
$$\delta p_f + \delta p_a = \frac{100 - 57}{200} = 0 \cdot 215 \quad . \quad . \quad . \quad (a)$$

As there is no added area in either body the levers h are indeterminate and the change in centroids of the fore and

after bodies will have to be used, i.e. $\delta \bar{x}_f$ and $\delta \bar{x}_a$. For these conditions:—

$$\phi_f \,.\, \delta \bar{x}_f = \phi_a \,.\, \delta \bar{x}_a \quad \text{[equation (37), section 6(c)]} \quad . \quad (b)$$

From equation (21) we have:—

$$\delta \bar{x} = \frac{-\delta p}{1-p}\left[\frac{(1-\phi)\,.\,B}{\phi} - (1 - 2\bar{x})\right]$$

Therefore

$$\delta \bar{x}_f = \frac{-\delta p_f}{1 - 0\cdot245}\left[\frac{(1 - 0\cdot7401)\,0\cdot7075}{0\cdot7401} - (1 - 2 \times 0\cdot3839)\right]$$

$$= -\,0\cdot0217\,\delta p$$

and

$$\delta \bar{x}_a = \frac{-\delta p_a}{1 - 0\cdot04}\left[\frac{(1 - 0\cdot6912)\,0\cdot6310}{0\cdot6912} - (1 - 2 \times 0\cdot3703)\right]$$

$$= -\,0\cdot0237\,\delta p_a$$

The shift of sections is given by equation (19), viz.:—

$$\delta x = \frac{\delta p\,(1-x)}{(1-p)}\left[1 - \frac{(1-\phi)(x-p)}{A}\right]$$

The shifts are therefore:—

Forebody:

$$\frac{0\cdot1085\,(1-x)}{(1-0\cdot245)}\left[1 - \frac{(1-0\cdot7401)(x-0\cdot245)}{0\cdot1082}\right] \times 200 \text{ ft.}$$

$$= 45\cdot8\,(1-x)\,(1 - 1\cdot513\,x) \text{ ft.}$$

Afterbody:

$$\frac{0\cdot1065\,(1-x)}{(1-0\cdot04)}\left[1 - \frac{(1-0\cdot6912)(x-0\cdot04)}{0\cdot167}\right] \times 200 \text{ ft.}$$

$$= 24\cdot0\,(1-x)\,(1 - 1\cdot728\,x) \text{ ft.}$$

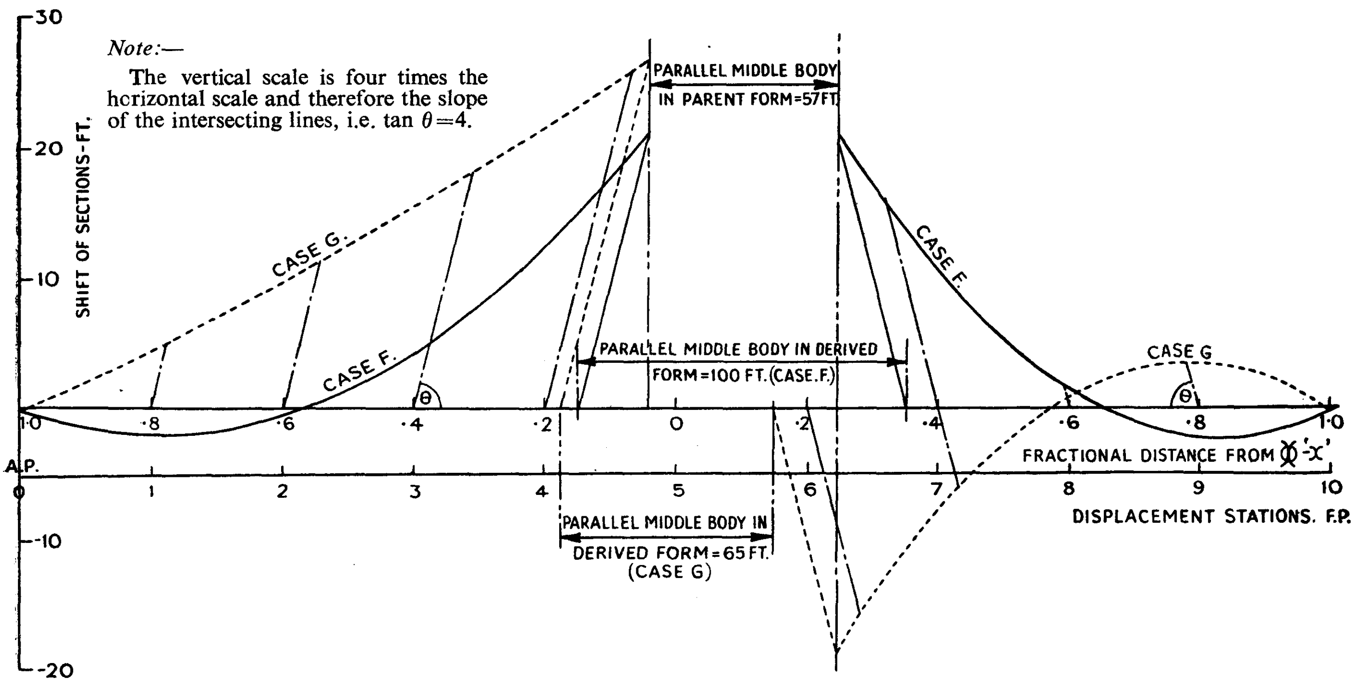

FIG. 16.—SHIFT OF SECTIONS FOR CASES F AND G

Case F.—Parallel middle body increased from 57 ft. to 100 ft.; block coefficient and LCB unaltered.
Case G.—Block coefficient increased by 0·025; LCB moved 2 ft. aft of original position; parallel middle in forebody reduced from 49 ft. to 30 ft.; parallel middle in afterbody increased from 8 ft. to 35 ft.

(*Note.*—For details see Appendix I, "Worked Examples.")

Substituting in equation (*b*) above we have:—

$$0\cdot7410 \times 0\cdot0217\,\delta p_f = 0\cdot6912 \times 0\cdot0237\,\delta p_a$$

Therefore $\delta p_f = 1\cdot02\,\delta p_a$.

Substituting in equation (*a*) above we have:—

$$1\cdot02\,\delta p_a + \delta p_a = 2\cdot02\,\delta p_a = 0\cdot215$$

Therefore $\delta p_a = 0\cdot1065$ and $\delta p_f = 0\cdot1085$.

Therefore new parallel middle in forebody $= 49 + (0\cdot1085 \times 200) = 70\cdot7$ ft.
and new parallel middle in afterbody $= 8 + (0\cdot1065 \times 200) = 29\cdot3$ ft.
} 100 ft. total.

These shifts are shown plotted on a contracted scale of length in Fig. 16 from which the position of the new sections can again be readily lifted as already described (*vide* section 7 and remarks under Case A in this Appendix). The positive shifts are *away* from midships and the negative shifts are *towards* midships.

Check on the Limits of the Variation

The practical limits are given by equation (23), viz.:—

$$\delta p = \frac{1-p}{1 \pm \dfrac{2\,(1-\phi)\,(1-p)}{A}}$$

Therefore

$$\text{Limit of } \delta\, p_f = \frac{(1 - 0\cdot245)}{1 + \dfrac{2\,(1 - 0\cdot7401)\,(1 - 0\cdot245)}{0\cdot1082}} = +\,0\cdot163$$

and

$$\text{Limit of } \delta\, p_a = \frac{(1 - 0\cdot04)}{1 + \dfrac{2\,(1 - 0\cdot6912)\,(1 - 0\cdot04)}{0\cdot1670}} = +\,0\cdot210$$

It will be noted that the variations of $\delta\, p$ deduced above are well within these limits.

Case G.—A general case will now be considered in which independent variation will be made of the following:—

(*a*) *Parallel middle in forebody: say reduce this by* 19 *ft., i.e.* $\delta\, p_f = -\,0\cdot095$.

(*b*) *Parallel middle in afterbody: increase this by* 27 *ft., i.e.* $\delta\, p_a = +\,0\cdot135$.

(*c*) *Block coefficient: increase this by* $0\cdot025$, *i.e.* $\delta\, \phi_t = +\,0\cdot0239$.

(*d*) *LCB: move this aft by* 2 *ft., i.e.* $\delta\, \bar{z} = -\,0\cdot01$.

(*Note.*—In this example general reference is made to section 5.)

$$\text{As before } \delta\, \phi_f = \frac{2\,[\delta\, \phi_t\,(h_a + \bar{z}) + \delta\, \bar{z}\,(\phi_t + \delta\, \phi_t)]}{h_f + h_a}$$

$$\text{and} \qquad \delta\, \phi_a = \frac{2\,[\delta\, \phi_t\,(h_f - \bar{z}) - \delta\, \bar{z}\,(\phi_t + \delta\, \phi_t)]}{h_f + h_a}$$

[equations (32) and (33), section 6)].

For this case the first approximations to the levers h for either body are given by equation (29), viz.:—

$$h = \phi \left\{ \frac{B}{\phi} \left[1 - \frac{\delta\, p\,(1 - \phi)}{\delta \phi\,(1 - p)} \right] + \frac{\delta\, p}{\delta\, \phi}\, \frac{(1 - 2\,\bar{x})}{(1 - p)} \right\}$$

It will be noted that, in this general case, the relations for h_f and h_a will themselves involve the required changes in fineness $\delta\, \phi_f$ and $\delta\, \phi_a$. This difficulty has been overcome by substituting the expressions for h_f and h_a in the equations (32) and (33) above and solving for $\delta\, \phi_f$ and $\delta\, \phi_a$.

These general expressions for $\delta\, \phi_f$ and $\delta\, \phi_a$ are:—

$$\delta\, \phi_f = \frac{2\,[\delta \phi_t (B_a + \bar{z}) + \delta \bar{z} (\phi_t + \delta \phi_t)] + C_f \,.\, \delta\, p_f - C_a \,.\, \delta\, p_a}{B_f + B_a} \qquad (40)$$

and

$$\delta \phi_a = \frac{2\,[\delta \phi_t (B_f - \bar{z}) - \delta \bar{z}\,(\phi_t + \delta \phi_t)] - C_f \,.\, \delta p_f + C_a \,.\, \delta p_a}{B_f + B_a} \qquad (41)$$

where A, B and C are the constants for the parent form referred to on page 15.

Making the necessary substitutions we have:—

$$\delta\, \phi_f = \frac{1}{0\cdot7075 + 0\cdot6310} \{2\,[0\cdot0239\,(0\cdot631 + 0\cdot0197) - 0\cdot01 \times 0\cdot7396] - 0\cdot016 \times 0\cdot095 - 0\cdot0162 \times 0\cdot135\}$$
$$= +\,0\cdot0094$$

and

$$\delta\, \phi_a = \frac{1}{0\cdot7075 + 0\cdot6310} \{2\,[0\cdot0239\,(0\cdot7075 - 0\cdot0197) + 0\cdot01 \times 0\cdot7396] + 0\cdot016 \times 0\cdot095 + 0\cdot0162 \times 0\cdot135\}$$
$$= +\,0\cdot0384$$

The shift of sections in each body is given by equation (28), viz.:—

$$\delta\, x = (1 - x) \left\{ \frac{\delta\, p}{(1 - p)} + \frac{(x - p)}{A} \left[\delta\, \phi - \delta\, p\, \frac{(1 - \phi)}{(1 - p)} \right] \right\}$$

The shifts in feet are therefore as follows:—

Forebody:

$$(1 - x) \left\{ \frac{-0\cdot095}{1 - 0\cdot245} + \frac{(x - 0\cdot245)}{0\cdot1082} \left[0\cdot0094 + 0\cdot095 \frac{(1 - 0\cdot7401)}{(1 - 0\cdot245)} \right] \right\}$$
$$\times\, 200 \text{ ft.} = 78\,(1 - x)\,(x - 0\cdot568) \text{ ft.}$$

Afterbody:

$$(1 - x) \left\{ \frac{0\cdot135}{1 - 0\cdot04} + \frac{(x - 0\cdot04)}{0\cdot1670} \left[0\cdot0384 - 0\cdot135 \frac{(1 - 0\cdot6912)}{(1 - 0\cdot04)} \right] \right\}$$
$$\times\, 200 \text{ ft.} = 5\cdot94\,(1 - x)\,(4\cdot77 - x) \text{ ft.}$$

These shifts are shown plotted on a contracted base of length in Fig. 16 from which the positions of the new displacement sections can be read off as already described (*vide* section 7 and remarks under Case A of this Appendix). The positive shifts are *away* from midships and the negative shifts are *towards* midships.

Check on the Limits of the Variation

The practical limits are given by equation (31), viz.:—

$$\delta\, \phi = \frac{\delta\, p\,(1 - \phi) \pm \tfrac{1}{2}\, A \left(1 - \dfrac{\delta\, p}{1 - p} \right)}{1 - p}$$

Therefore the limits of $\delta\, \phi_f$

$$= \frac{-\,0\cdot095\,(1 - 0\cdot7401) \pm \tfrac{1}{2} \times 0\cdot1082 \left(1 + \dfrac{0\cdot095}{1 - 0\cdot245} \right)}{1 - 0\cdot245}$$

$$= +\,0\cdot0478 \text{ or } -\,0\cdot1133$$

and the limits of $\delta\, \phi_a$

$$= \frac{0\cdot135\,(1 - 0\cdot6912) \pm \tfrac{1}{2} \times 0\cdot1670 \left(1 - \dfrac{0\cdot135}{1 - 0\cdot04} \right)}{1 - 0\cdot04}$$

$$= +\,0\cdot1182 \text{ or } -\,0\cdot0313.$$

It will be seen that the variations in $\delta\, \phi_f$ and $\delta \phi_a$ are well within these limits.

The comprehensive relations used in this example could equally have been applied to the previous examples C, D, E and F, which are in effect particular cases of this general case. This would mean putting one or more of the changes $\delta\, \phi_t$, $\delta\, \phi_f$, $\delta\, \phi_a$, $\delta\, \bar{z}$, $\delta\, p_f$ and $\delta\, p_a$ equal to nought as appropriate to suit the particular case and, as a matter of interest, the previous examples C, D, E and F have been checked in this manner. It will be noted that the general equations (40) and (41) for $\delta\, \phi_f$ and $\delta\, \phi_a$ are not suitable for applying the "one minus prismatic" variation as $\delta\, p_f$ and $\delta\, p_a$ will depend upon $\delta\, \phi_f$ and $\delta\, \phi_a$. Methods of carrying out this particular variation are shown in the worked examples A and B.

Experience may show that it is generally more convenient to use the comprehensive relations given in this example, and in this connection it is suggested that the form constants A, B, and C, which depend only on the geometrical properties

of the original form, should be worked out as a matter of course for good parent ships generally used as bases for new designs. Any desired variation could then be rapidly applied. For convenience the necessary general relations are summarized below.

Summary of the General Relations

$$\delta\phi_f = \frac{2\,[\delta\phi_t\,(B_a + \bar{z}) + \delta\,\bar{z}\,(\phi_t + \delta\,\phi_t)] + C_f\,.\,\delta p_f - C_a\,.\,\delta\,p_a}{B_f + B_a} \quad . \quad . \quad . \quad . \quad . \quad (40)$$

$$\delta\,\phi_a = \frac{2[\delta\,\phi_t\,(B_f - \bar{z}) - \delta\,\bar{z}(\phi_t + \delta\,\phi_t)] - C_f\,.\,\delta p_f + C_a\,.\,\delta p_a}{B_f + B_a} \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The items in the following expressions refer to either the fore or after bodies as appropriate.
Shift of sections in either half-body:—

$$\delta\,x = (1 - x)\left\{\frac{\delta\,p}{1 - p} + \frac{(x - p)}{A}\left[\delta\,\phi - \delta\,p\,\frac{(1 - \phi)}{(1 - p)}\right]\right\} \,.\, (28)$$

Practical limits of $\delta\,\phi_f$ and $\delta\,\phi_a$:—

$$\delta\,\phi = \frac{\delta\,p\,(1 - \phi) \pm \frac{1}{2}\,A\left(1 - \dfrac{\delta\,p}{1 - p}\right)}{1 - p} \quad . \quad . \quad (31)$$

In these equations:—

$$A = \phi\,(1 - 2\,\bar{x}) - p\,(1 - \phi)$$

$$B = \frac{\phi\,[2\,\bar{x} - 3\,k^2 - p\,(1 - 2\,\bar{x})]}{A}$$

$$C = \frac{B\,(1 - \phi) - \phi\,(1 - 2\,\bar{x})}{1 - p}$$

The various items involved in the form constants A, B, and C above can be conveniently calculated in the manner indicated at the beginning of this Appendix.

Appendix II

DERIVATION OF THE VARIOUS RELATIONS GIVEN IN THE PAPER

(i) The "One Minus Prismatic" Variation

Referring to Fig. 1 and section 2 on p. 2.
The shift of sections is given by equation (1), viz.:—

$$\delta\,x = \frac{\delta\,\phi}{(1 - \phi)}\cdot(1 - x)$$

The moment about midships of the added "sliver" of area $\delta\,\phi$ is given by:—

$$\delta\,\phi\,.\,h = \int_0^1 \delta\,x\left(x + \frac{\delta\,x}{2}\right)d\,y = \int_0^1 \delta\,x\,.\,x\,.\,d\,y + \tfrac{1}{2}\int_0^1 \delta\,x^2\,.\,d\,y$$

Substituting for $\delta\,x$ from equation (1),

$$\delta\,\phi\,.\,h = \frac{\delta\,\phi}{1 - \phi}\int_0^1 (x - x^2)\,d\,y + \frac{\delta\,\phi^2}{2(1 - \phi)^2}\int_0^1 (1 - 2\,x + x^2)\,d\,y$$

But $\displaystyle\int_0^1 x\,d\,y = \phi$; and $\displaystyle\int_0^1 x^2\,d\,y = 2\,\phi\,.\,\bar{x}$

Therefore

$$\delta\,\phi\,.\,h = \frac{\delta\,\phi}{1 - \phi}(\phi - 2\,\phi\,.\,\bar{x}) + \frac{\delta\,\phi^2}{2\,(1 - \phi)^2}(1 - 2\,\phi + 2\,\phi\,.\,\bar{x})$$

Therefore the lever h

$$= \frac{\phi\,(1 - 2\,\bar{x})}{1 - \phi} + \frac{\delta\,\phi}{2\,(1 - \phi)^2}[1 - 2\,\phi\,(1 - \bar{x})] \text{ (equation 3)}$$

If $\delta\,\phi$ is small compared with ϕ then the first approximation to h is given by $\dfrac{\phi\,(1 - 2\,\bar{x})}{(1 - \phi)}$ (equation 4).

(ii) Varying the Fullness of an Entrance or Run not associated with Parallel Middle Body

Referring to Fig. 2 and section 3 on p. 3. In this case there is no parallel middle body in either the basis or the derived form and the length of entrance (or run) is identical with the length of the half-body.

(*a*) *Shift of Sections* $\delta\,x$

The required terminal conditions are obtained by using a relation between $\delta\,x$ and x of the form:—

$$\delta\,x = c\,.\,x\,(1 - x), \text{ where } c \text{ is a constant.}$$

The problem now is to determine the value of c in terms of the change in fineness $\delta\,\phi$.

$$\delta\,\phi = \int_0^1 \delta\,x\,.\,d\,y = c\int_0^1 x\,(1 - x)\,d\,y = c\int_0^1 (x - x^2)\,d\,y$$

i.e. $$\delta\,\phi = c\int_0^1 x\,d\,y - c\int_0^1 x^2\,d\,y = c\,(\phi - 2\,\phi\,.\,\bar{x})$$

Hence $$c = \frac{\delta\,\phi}{\phi\,(1 - 2\,\bar{x})}$$

Therefore $$\delta\,x = \frac{\delta\,\phi\,.\,x\,(1 - x)}{\phi\,(1 - 2\,\bar{x})} \text{ (equation 5)}$$

(*b*) *Distance from Midships of the Centroid of the Added Area* $\delta\,\phi$

The moment of the added "sliver" of area about midships

$$\delta\,\phi\,.\,h = \int_0^1 \delta\,x\left(x + \frac{\delta\,x}{2}\right)d\,y$$

$$= \int_0^1 \delta\,x\,.\,x\,.\,d\,y + \tfrac{1}{2}\int_0^1 \delta\,x^2\,.\,d\,y$$

Substituting for $\delta\,x$ from equation (5) we have:—

$$\delta\,\phi\,.\,h = \frac{\delta\,\phi}{\phi\,(1 - 2\,\bar{x})}\int_0^1 (x^2 - x^3)\,d\,y + \frac{\delta\,\phi^2}{2\,\phi^2\,(1 - 2\,\bar{x})^2}\int_0^1 (x^2 - 2\,x^3 + x^4)\,d\,y$$

but $\displaystyle\int_0^1 x^3\,d\,y = 3\,\phi\,.\,k^2$ and $\displaystyle\int_0^1 x^4\,.\,d\,y = 4\,\phi\,.\,r^3$

where k is the radius of gyration (or lever of the second moment) of the original area curve about midships and r is the lever of the third moment about midships;

i.e. $$\delta\phi\,.\,h = \frac{\delta\phi}{\phi(1-2\,.\,\bar{x})}\Big[(2\,\phi\,.\,\bar{x} - 3\,\phi\,.\,k^2) + \frac{\delta\phi}{2\,\phi(1-2\,\bar{x})}(2\,\phi\,.\,\bar{x} - 6\,\phi\,.\,k^2 + 4\,\phi\,.\,r^3)\Big]$$

Therefore

$$h = \frac{2\,\bar{x} - 3\,k^2}{1 - 2\,\bar{x}} + \frac{\delta\phi}{\phi}\cdot\frac{(\bar{x} - 3\,k^2 + 2\,r^3)}{(1-2\,\bar{x})^2} \quad \text{(equation 7)}$$

If $\delta\phi$ is small compared with ϕ then the first approximation to h is given by:—

$$h = \frac{2\,\bar{x} - 3\,k^2}{1 - 2\,\bar{x}} \quad \text{(equation 6)}$$

(*c*) *Limiting Conditions*

$\delta x = c\,.\,x\,(1 - x)$ as before.

Let x' = new abcissa of derived curve = $x + \delta x$, i.e. $x' = x + c\,.\,x\,(1 - x)$.

Differentiate with respect to y and we have

$$\frac{d\,x'}{d\,y} = \frac{d\,x}{d\,y} + c\frac{d\,x}{d\,y} - 2\,c\,.\,x\,.\frac{d\,x}{d\,y}$$

Therefore $$\frac{d\,x'}{d\,y} = \frac{d\,x}{d\,y}[1 + c\,(1 - 2\,x)]$$

or $$\frac{d\,y}{d\,x'} = \frac{d\,y}{d\,x}\cdot\frac{1}{1 + c\,(1 - 2\,x)}$$

This gives the relation between the slope of the derived curve and the slope of the basis curve at the same ordinate y.

At the shoulder $x = 0$ and we have:—

$$\frac{d\,y}{d\,x'} = \frac{d\,y}{d\,x}\cdot\frac{1}{1 + c}$$

The slope of the basis curve $(d\,y/d\,x)$ will be zero here and consequently the slope of the derived curve $(d\,y/d\,x')$ will also be zero provided c is greater than -1. This latter condition gives the lower limit for $\delta\phi$.

That is $$c = \frac{\delta\phi}{\phi\,(1 - 2\,\bar{x})} = -1$$

Therefore $$\delta\phi = -\phi\,(1 - 2\,\bar{x})$$

At this limiting value of $\delta\phi$ the derived curve is not tangential to the half-breadth, but generally forms an angular shoulder (*vide* section 9 and Figs. 10 and 11).

At $x = 1$ we have:—

$$\frac{d\,y}{d\,x'} = \frac{d\,y}{d\,x}\cdot\frac{1}{1 - c}$$

If the slope of the original curve $(d\,y/d\,x)$ is finite here, then the slope of the derived curve $(d\,y/d\,x')$ will also be finite provided c is less than $+1$. This condition gives the upper limit for $\delta\phi$, i.e. $\delta\phi = +\phi\,(1 - 2\,\bar{x})$ is the absolute upper limit of the variation. If the slope of the basis curve is zero at this point, the slope of this limiting derived curve is indeterminate from the above relation, but experience shows that it approaches the base line at a very steep angle.

The absolute limits of $\delta\phi$ are therefore given by:—

$$\delta\phi = \pm\,\phi\,(1 - 2\,\bar{x}) \quad \text{(equation 8)}$$

Outside of these limits the derived curves would come outside of the boundaries of the half-body.

(iii) **General Case**

Referring to Fig. 7, section 5 on p. 6.

(*a*) *Shift of Sections* δx

To be able to satisfy the terminal conditions it will be necessary to use an expression for δx of the form

$$\delta x = c\,(1 - x)\,(x + d)$$

where c and d are constants in a given case.

Terminal conditions:—

At $x = 1$, $\delta x = 0$; at $x = p$, $\delta x = \delta p$.

The condition at $x = 1$ is clearly satisfied; with regard to $x = p$ we have:—

$$\delta p = c\,(1 - p)\,(p + d)$$

i.e. $$d = \frac{\delta p}{c\,(1 - p)} - p$$

Hence $$\delta x = c\,(1 - x)\,(x - p) + \frac{\delta p}{(1 - p)}(1 - x)$$

and

$$\delta\phi = \int_0^1 \delta x\,d\,y$$
$$= c\int_0^1 (x - x^2 - p + p\,.\,x)\,d\,y + \frac{\delta p}{1 - p}\int_0^1 (1 - x)\,d\,y$$
$$= c\,(\phi - 2\,\phi\,.\,\bar{x} - p + p\,.\,\phi) + \frac{\delta p}{1 - p}(1 - \phi)$$

Therefore $$c = \frac{\delta\phi - \delta p\dfrac{(1 - \phi)}{(1 - p)}}{\phi\,(1 - 2\,\bar{x}) - p\,(1 - \phi)}$$

Hence

$$\delta x = (1 - x)\left\{\frac{\left[\delta\phi - \delta p\dfrac{(1 - \phi)}{(1 - p)}\right]}{\phi\,(1 - 2\,\bar{x}) - p\,(1 - \phi)}\cdot(x - p) + \frac{\delta p}{(1 - p)}\right\}$$

Substituting A for $\phi\,(1 - 2\,\bar{x}) - p\,(1 - \phi)$ we have equation (28), viz.:—

$$\delta x = (1 - x)\left\{\frac{\delta p}{1 - p} + \frac{(x - p)}{\text{A}}\left[\delta\phi - \delta p\frac{(1 - \phi)}{(1 - p)}\right]\right\}$$

(*b*) *Distance from Midships of the Centroid of the Added Area* $\delta\phi$

The moment of the added "sliver" of area about midships is given by:—

$$\delta\phi\,.\,h = \int_0^1 \delta x\,.\,x\,.\,d\,y \quad \text{(first approximation)}$$

i.e. $$\delta\phi\,.\,h = c\int_0^1 (x^2 - x^3 - p\,.\,x + p\,.\,x^2)\,d\,y + \frac{\delta p}{(1 - p)}\int_0^1 (x - x^2)\,d\,y$$
$$= c\,(2\,\phi\,.\,\bar{x} - 3\,\phi\,.\,k^2 - p\,.\,\phi + 2\,p\,.\,\phi\,.\,\bar{x}) + \frac{\delta p}{(1 - p)}(\phi - 2\,\phi\,.\,\bar{x})$$

$$= c \, . \, \phi \, [2 \, \bar{x} - 3 \, k^2 - p \, (1 - 2 \, \bar{x})] + \frac{\delta \, p \, . \, \phi}{(1 - p)} (1 - 2 \, \bar{x})$$

Substituting for c and dividing by $\delta \, \phi$ we have the lever

$$h = \phi \left\{ \frac{\left[1 - \frac{\delta \, p \, (1 - \phi)}{\delta \, \phi \, (1 - p)}\right]}{\phi \, (1 - 2 \, \bar{x}) - p \, (1 - \phi)} [2 \, \bar{x} - 3 \, k^2 - p \, (1 - 2 \, \bar{x})] + \frac{\delta \, p}{\delta \, \phi} \cdot \frac{(1 - 2 \, \bar{x})}{(1 - p)} \right\}$$

Substituting B for $\dfrac{\phi \, [2 \, \bar{x} - 3 \, k^2 - p \, (1 - 2 \, \bar{x})]}{\phi \, (1 - 2 \, \bar{x}) - p \, (1 - \phi)}$ we have equation (29) viz.:—

$$h = \phi \left\{ \frac{\mathrm{B}}{\phi} \left[1 - \frac{\delta \, p \, (1 - \phi)}{\delta \, \phi \, (1 - p)}\right] + \frac{\delta \, p}{\delta \, \phi} \frac{(1 - 2 \, \bar{x})}{(1 - p)} \right\}$$

(*c*) *Limiting Conditions*

$\delta \, x = c \, (1 - x) \, (x + d)$ and as before let $\bar{x}'$ = abcissa of derived curve $= x + \delta \, x$.

Then $\quad x' = x + c \, (1 - x) \, (x + d)$
$\qquad\quad = x + c \, (x - x^2 + d - d \, . \, x)$

Differentiate with respect to y and we have

$$\frac{d \, x'}{d \, y} = \frac{d \, x}{d \, y} [1 + c \, (1 - 2 \, x - d)]$$

and
$$\frac{d \, y}{d \, x'} = \frac{d \, y}{d \, x} \cdot \frac{1}{1 + c \, (1 - 2 \, x - d)}$$

This gives the slope of the derived curve in terms of that of the basis curve at the same ordinate y.

At the shoulder $x = p$ and we have:—

$$\frac{d \, y}{d \, x'} = \frac{d \, y}{d \, x} \cdot \frac{1}{1 + c \, (1 - 2 \, p - d)}$$

The slope of the basis curve $(d \, y / d \, x)$ will be zero here and the slope of the derived curve $(d \, y / d \, x')$ will also be zero provided $c \, (1 - 2 \, p - d)$ is greater than $- 1$.

The limiting case is given by:—

$$c \, (1 - 2 \, p - d) = - 1$$

Substituting for d we have:—

$$c \, (1 - 2 \, p) - \frac{\delta \, p}{1 - p} + c \, . \, p = - 1$$

i.e.
$$c \, (1 - p) = - \left(1 - \frac{\delta \, p}{1 - p}\right)$$

Substituting for c we have:—

$$\frac{\delta \, \phi \, (1 - p) - \delta \, p \, (1 - \phi)}{\phi \, (1 - 2 \, \bar{x}) - p \, (1 - \phi)} = - \left(1 - \frac{\delta \, p}{1 - p}\right)$$

Putting $\mathrm{A} = \phi \, (1 - 2 \bar{x}) - p \, (1 - \phi)$ we have

the lower limit of
$$\delta \, \phi = \frac{\delta \, p \, (1 - \phi) - \mathrm{A}\left(1 - \frac{\delta \, p}{1 - p}\right)}{1 - p}$$

At $x = 1$ we have:—

$$\frac{d \, y}{d \, x'} = \frac{d \, y}{d \, x} \cdot \frac{1}{1 - c \, (1 + d)}$$

If the slope of the original curve is finite here, then the slope of the derived curve will also be finite provided $c \, (1 + d)$ is less than $+ 1$. This condition gives the upper limit for $\delta \, \phi$.

The limiting case is given by:—

$$c \, (1 + d) = 1$$

Substituting for d we have:—

$$c + \frac{\delta \, p}{1 - p} - c \, . \, p = 1$$

Therefore
$$c \, (1 - p) = 1 - \frac{\delta \, p}{1 - p}$$

Substituting for c we have:—

$$\frac{\delta \, \phi \, (1 - p) - \delta \, p \, (1 - \phi)}{\phi \, (1 - 2 \, \bar{x}) - p \, (1 - \phi)} = 1 - \frac{\delta \, p}{1 - p}$$

Again putting $\mathrm{A} = \phi \, (1 - 2 \, \bar{x}) - p \, (1 - \phi)$ the upper limit for $\delta \, \phi$ is given by:—

$$\delta \, \phi = \frac{\delta \, p \, (1 - \phi) + \mathrm{A}\left(1 - \frac{\delta \, p}{1 - p}\right)}{1 - p}$$

The absolute limits for $\delta \, \phi$ in the general case are therefore given by:—

$$\delta \, \phi = \frac{\delta \, p \, (1 - \phi) \pm \mathrm{A}\left(1 - \frac{\delta \, p}{1 - p}\right)}{(1 - p)} \quad \text{(equation 30)}$$

As explained in the text (section 5 on p. 6) all the special cases given in the paper can be derived from the general equations given above by substitution of the appropriate conditions of parallel middle body, etc.

Appendix III

CHANGING POSITION OF LCB BY "SWINGING" THE AREA CURVE

For the sake of completeness an account is given below of the well-known method of changing the LCB position of a ship's form by "swinging" the area curve, the fullness being maintained constant.

In Fig. 17 the full line A B C represents the complete sectional area curve for the basis ship and the dotted line the derived curve having the new position of LCB.

In this method of form variation the "shift" $(\delta \, x)$ of any particular section x is made proportional to the ordinate y of the area curve. That is, in the notation adopted in the paper, $\delta \, x = c \, . \, y$ where c is a constant depending on the required movement of the LCB. In effect, therefore, each ordinate is "swung" through the same angle θ as shown and

$$\tan \theta = \frac{\delta \, x}{y} = c, \text{ or } \delta \, x = y \, . \tan \theta$$

The shift of sections is in the same direction in both the

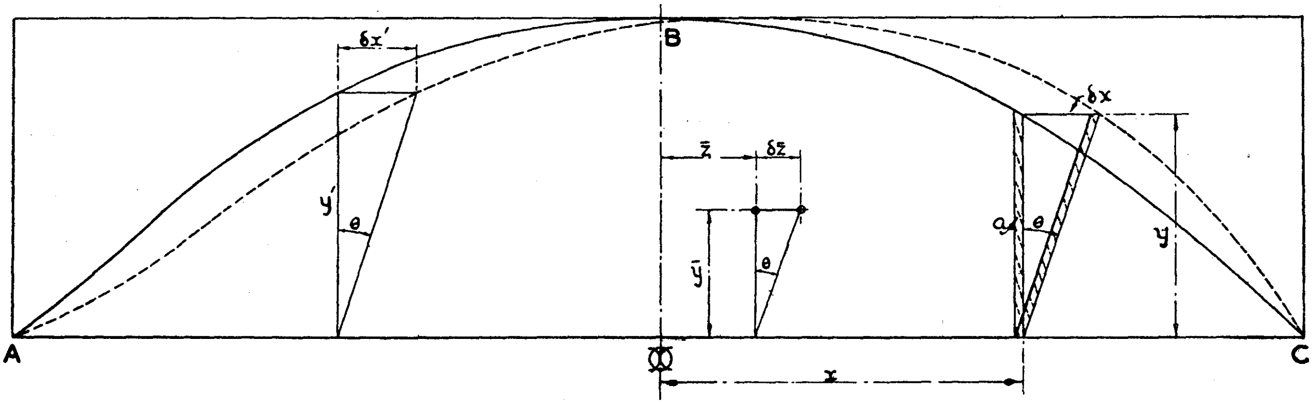

FIG. 17

fore and after bodies, depending on which way the LCB is to be moved.

Let $\bar{z}$ = the position of LCB in the original form,
$\delta \bar{z}$ = the required change in LCB position,
A = the total area under the original curve A B C,
$\bar{y}$ = the position of the vertical centroid of the original area A above the base.

Considering a thin vertical strip of area "a" as shown, the longitudinal transfer of moment due to swinging $= a \,.\, \frac{1}{2} \delta x = a \,.\, \frac{1}{2} y \tan \theta$. The total transfer of moment for the whole form will be the summation of this for all such strips, viz. $\tan \theta \, \Sigma \, a \,.\, \frac{1}{2} y$; but $\Sigma \, a \,.\, \frac{1}{2} y$ is the moment of the original area curve about the base which equals $A \,.\, \bar{y}$. The total longitudinal transfer of moment is therefore given by $A \,.\, \bar{y} \,.\, \tan \theta$.

This total transfer of moment is also given by $A \,.\, \delta \bar{z}$, therefore:—

$A \,.\, \bar{y} \,.\, \tan \theta = A \,.\, \delta \bar{z}$ and therefore $\tan \theta = \frac{\delta \bar{z}}{\bar{y}} = c$, the constant referred to above.

The shift of sections is thus given by:—

$$\delta x = \frac{\delta \bar{z}}{\bar{y}} \cdot y \quad . \quad . \quad . \quad . \quad . \quad (42)$$

There is no particular advantage to be gained in using "fractional" shifts in this variation; the units of $\bar{y}$ should be the same as those of y; δx will then be in the same units as $\delta \bar{z}$.

The curve of shift of sections is therefore simply the original area curve drawn to a different vertical scale. By adjusting this vertical scale as necessary, the positions of the new displacement sections on the original form can be readily scaled off as described in section 7 of the paper and the offsets for the new body sections can then be lifted directly from the half-breadth plan of the basis ship.

It will be seen that, as in the "one minus prismatic" variation, there is no control over the position of the parallel middle body (or maximum section). This is pushed bodily either forward or aft as the case may be. This bodily shift is given by $\frac{\delta \bar{z}}{\bar{y}} \,.\, Y$, where Y is the maximum ordinate of the area curve.

DISCUSSION

Mr. J. L. Kent, C.B.E. (Member): Mr. Lackenby's paper is an admirably clear exposition of the methods which can be employed to make a geometrical change from a parent form of ship to satisfy some desired change in dimension or distribution of displacement, etc. The methods he proposes are accurate, practical, easily understood, and, by abolishing trial and error, should save much time and labour in the design room.

When a series of models are derived from a good parent form by a systematic change in one or more dimensions, the user generally expects each model of the series to be the best possible for resistance. Assuming the parent form to be the best possible, if a geometrical change to rule is made and pushed to the limit, somewhere in the series of models a sudden rise in resistance will occur, and the shapes of the forms of hulls beyond this point are usually not the best possible for the dimensions and speed. The rise in resistance is caused by the violation of some natural law, such as increasing the slope of the after lines beyond the limit where breakdown in stream flow occurs, or diminishing the prismatic length of the entrance below the wave-length appropriate to the ship's speed, or to some other hydrodynamic law, either unknown or imperfectly understood. I would suggest that, when a series of model hulls are to be made for an experimental investigation into the effect upon the resistance of a regular change in one or more dimensions, the author's methods of geometrical change from the parent form are adopted. But before deciding upon the lower and upper limits to which the series is taken, I would further suggest that such items as limiting slopes of both fore and after level lines, limiting fullnesses of entrance and run, etc., should be considered, as these are not determined in the author's formulae. Perhaps he could indicate the geometric limits of these critical slopes and fullnesses in his formulae. Further, I would urge the importance of model experiments aimed at discovering the critical limits of tangential slope, minimum displacement lengths, etc., and the discovery of the reasons which cause these to be critical limits of form for resistance.

Mr. J. Thomas Tothill, B.Sc. (Associate-Member): The need for a systematic method of varying ship forms arises whenever a methodical series of model tests is being planned. In the past a variety of methods have

been developed for this purpose, but the details have rarely been published. The present paper, offering an exact method of varying the prismatic, parallel body, and LCB in any combination without the necessity of re-fairing the lines, is all the more welcome for the scarcity of comparable information. The details of the method, including the limits within which it can be usefully applied, are most competently treated and the examples give a clear picture of its versatility.

A further use of the method not considered in the paper is that it can equally well be applied to the curve of water-line areas to vary the vertical prismatic and VCB in a consistent manner. It also has a restriction which is not mentioned, and that is that it cannot be applied to a vessel with a raked keel without buckling the keel line.

Figs. 10 and 11 bear a marked resemblance to the type of variation used by Taylor in the Standard Series, and it is distinctly probable that he may have used a very similar and possibly identical method. Mr. Lackenby's method now offers the possibility of extending Taylor's work in the directions of variable parallel body and variable LCB. If a new series were run to investigate these variables over a broad range, it would be very desirable to use the same parent form that Taylor used in the Standard Series so that a tie-in would be possible.

As the author states, there is no particular reason, apart from keeping the mathematical relations simple, for choosing a parabolic curve of sectional shifts of the type $x(1-x)$. For certain applications it might well be found that a more flexible form, such as the one suggested in the paper, would be preferable. For the great majority of applications, however, the examples given in the paper suggest that the simpler form adopted by Mr. Lackenby will give very satisfactory results.

Mr. F. W. Benson (Member): There are very few who have to do with ships who do not also take an interest in ships' lines. The writer has been intrigued with the neat way the author has led up by particular cases, in which a basis form can be varied, to the general case (section 5 and Appendix II).

It is very useful to have all these methods of varying ships' lines under one cover and I am sure that this will be appreciated.

Appendix II is attractive and should be instructive to the design draughtsman. Here the mathematical representation of the operations that are performed daily by means of Simpson's rules, the planimeter and the integrator are set out in a way that is bound to lead to a better understanding of what is actually being done by means of these aids, for instance, $\int_0^1 x^3\,dy$ is equal to three times the moment of inertia of a rectangle about one end $3\,\phi\,k^2$ and so on, of course in this case a very narrow rectangle.

In dealing with the upper and lower limits to which a basis sectional area curve can be varied, page 297, it would appear that half the absolute limit plus or minus is well within the practical range, and that although a little fairing would have to be done at the shoulder this would not appreciably affect the results. The writer has verified this for a few cases. Theoretically, of course, when a curve joins a straight line it should not only be tangent with it but also have the same radius of curvature at the joining point.

Fig. 9 of section 7 is ingenious and it would be difficult to conceive any other way of getting an accurate derived body plan with so little work.

The writer has not tried out in detail how accurate the first approximation to "h" could be in all cases (equations 4 and 6), say, for instance, when $\delta\,\phi$ is 15 per cent of ϕ. On page 291 the author states that when the change in ϕ is moderate experience shows that the second term of equation (3) is negligible compared with the first, and that "h," as given by equation (4), would be a good first approximation. No doubt this may be the case, but could the author state what the value of both terms of equation (3) would be if $\delta\,\phi$ were 15 per cent of ϕ, calculated from the parent curve, Fig. 11.

Mr. H. Bocler (Member): In stating and developing different methods by which ship lines can be systematically varied the author has performed a considerable service to those who have in practice to design the form of a new ship. The author's investigations have had as their main purpose the derivation of forms for methodical series of resistance experiments on ship models, but there are other directions in which systematic variation of a basic form is of practical usefulness in the preliminary estimates for the various requirements in a new ship. By such means control can be kept of displacement, cubic capacity and moment of inertia of water plane in relation to transverse stability and longitudinal trim, without recourse to long detail calculations until a stage approaching finality has been reached. It must, however, always be borne in mind that a form derived by any methods of variation found thus convenient should be examined on its own merits before final acceptance.

Mr. T. Corin (Associate-Member): The author has investigated a standard design office problem and has produced solutions which exhibit a high standard of mathematical cunning. Papers of this type are slowly transforming our subject from an art into a science.

The older solutions were unsatisfactory in many cases. "Swinging" the area curve always introduced the "now-routine" calculation of the vertical centroid. When design requirements were quite close to those of a basis ship, however, the "one minus prismatic" solution was quite satisfactory. A desired LCB was not difficult to achieve when suitable differences were made to the fore- and after-body prismatic coefficients. Such differences have been given by Mr. E. E. Bustard in his paper "Preliminary Calculations in Ship Design" (*Trans. N.E.C.*, Vol. LVII).

The author's method is particularly suited to deal with systematic series, with forms without parallel

middle-body and where no close basic data are available. For hollow bows, some naval architects place stress on the position of the point of inflection in the area curve and this appears to be the one remaining variable not covered by the paper. In several places it has been implied that new lines need not necessarily be drawn, but that displacement sections can be lifted off at a new spacing. This will not be quite true near the bilge if a change in beam has been made. Further, caution will be necessary where large changes in basic particulars have been made. It is well known that the $\frac{1}{2}$-angle of entrance of the W.L. is important and only actual experience will show whether shifts of sections using this new method will produce reasonable waterline forms.

Dr. G. P. Weinblum: The paper is indicative of the need for introducing some kind of system when dealing with ship lines.

The "one minus prismatic" variation is, of course, an application of the well-known affine transformation, which is constantly used in naval architecture. Formulae (1) to (4) will prove to be useful since they describe familiar procedures in a rigorous way.

The application of the affine transformation is obviously very limited when making systematic variations. Therefore, the author proposes some new transformations which lead to interesting relations between important parameters. This is a useful but not sufficient result. The present writer holds the opinion that the whole approach suffers somewhat from a habit, quite common in naval architecture, of developing a kind of special mathematics peculiar to the profession. The problem can be attacked in a more orthodox and at the same time more radical way.

The author's proposal consists essentially in a variation of curves by varying the *abscissa* (independent variable), which leads to rather cryptic equations of the resulting curve. The transforming function is, nevertheless, very special. The present writer suggests using if necessary only systematic variation of the abscissa in the affine transformation, because of its basic importance and simplicity, and applying all other variations to the ordinates, i.e. to vary the equation $y = f(x)$ of the curve. When starting with an analytical equation of the parent line this approach appears to be obvious, but it keeps its merits when for some reasons the parent line is given only graphically, although it is always possible to approximate such a line within the accuracy needed by simple functions, preferably polynomials. Fundamental work on mathematical ship lines has been done by D. W. Taylor and his results have been extended by other authors.

Our present trend in using mathematical expressions for varying parent forms has two principal targets:—

(1) To fix the forms used in a definite way by equations admitting a high generality.

(2) To use mathematical expressions which are suitable tools for calculating resistance, oscillations in a seaway, etc.

As far as the present writer can see, the author's proposal complies only to a certain degree with the first postulate and in a very limited way with the second.

Dr. G. Hughes (Member): The author has covered his subject in a very comprehensive manner and has produced a valuable survey of systematic geometrical variations of ship lines to which frequent reference will be made.

The greater part of the paper is devoted to the development of the more general methods of form derivation proposed by the author which may be said to be covered by $\delta x = c x^n (1 - x^m)$, together with adjustments of parallel middle body. Attention has been concentrated on the more particular case given by $\delta x = c x (1 - x)$, which leads to solutions which are not too unwieldy. The maximum shift occurs at $x = \frac{1}{2}$, whereas with the "one minus prismatic" variation the maximum shift is at the shoulder. It is felt that in practice derivation of one design from another tends to be somewhat between these limits, such as might be covered by $\delta x = c x (1 - x)^n$, which has a maximum value at $x = \dfrac{1}{n+1}$. Thus if $n = 2$, the maximum shift is at one-third, and if $n = 3$, at one-fourth, and so on, of the length from shoulder to end. This form appears to lead to simpler initial solutions than the form $\delta x = c x^n (1 - x^m)$, and if developed might prove to be of practical value. Possibly the author has considered this variation; if not, it is suggested that its possibilities should be explored.

Mr. W. W. Weyndling, B.Sc. (Associate-Member): It may be of some interest that the methods of systematic geometrical variation of ship forms outlined in this paper found application in a shipyard design office* attempt at putting to every-day use the B.S.R.A. methodical series tank results in conjunction with the available shipyard tank data.

Assuming that in normal "from scratch" design work block coefficient is fixed by the design speed and carrying capacity required, it was thought that the position of LCB and the size of bilge radius could perhaps be taken as the two basic form characteristics which remain to be determined. It was further thought that with the help of the B.S.R.A. methodical series results the effect could be reasonably predicted of bilge radius and LCB variations of a parent form suitably chosen from the firm's tank-tested designs on its Ⓒ values, provided the resulting alterations in shape of the area curve followed reasonably closely those of the B.S.R.A. standard form. The methods of geometrical form variation outlined in the paper allowed any degree of similarity to be approached in re-distribution of area under the curve along the ship's length.

A number of naked hull resistance results for tank tested designs of block coefficients close to the B.S.R.A. $0{\cdot}75\ C_b$ standard form were reduced to $0{\cdot}75\ C_b$ and 400-ft. length using respectively Ayre's and Baker's

* Furness Shipbuilding Co. Ltd.

corrections to Ⓒ values. The Ⓒ curves were then compared and the most suitable design chosen as a $0 \cdot 75\ C_b$ parent form.

Sets of Ⓒ curves were then plotted on the base of LCB position for different bilge radii. Each set corresponded to a $V/\sqrt{L}$ value, taken from the range of $0 \cdot 40$ to $0 \cdot 70$ at suitable intervals, and to a draught of 21 or 26 ft. It was intended to produce similar sets of curves for parent forms corresponding to different standard block coefficient forms in forthcoming B.S.R.A. methodical series. Thus it would be possible to choose for a new design of fixed C_b almost at sight the most efficient bilge radius and position of LCB (or one or the other if one of them were given) for the required range of $V/\sqrt{L}$, from the point of view of naked resistance, and also to find the order of the corresponding Ⓒ value.

It was now necessary to give effect to the changes in shape of the area curve (or re-distribution of prismatic coefficient along ship's length) resulting from the bilge radius and LCB variations in a manner closely resembling that of the B.S.R.A. series.

The variation of the position of LCB was easily effected by the "swinging" method.

The bilge radius variations for a given position of LCB and while maintaining constant C_b carried out in the B.S.R.A. series, require adjustments of prismatic and are more difficult to follow. The conditions defining the variation of prismatic can be taken as shown in the general case of the paper:—

$$\delta x = 0, \text{ at } x = 1$$
$$\text{and } \delta x = \delta p, \text{ at } x = p$$

An equation of the type $\delta x = c\,(1 - x)^m\,(x + d)$ satisfying those conditions has to be chosen, so as to suit the desired concentration of $\delta \phi$. Usually an equation where $m = 1$, 2, or 3 will allow the distribution of area to be controlled to any desired extent.

Taking an equation of the form:

$$\delta x = c\,(1 - x)^2\,(x + d)$$

As $\delta \phi$ and δp are already fixed, the former automatically by the change in bilge radius, the latter by the desired alteration in parallel middle body, the constants "c" and "d" can be found simply from the simultaneous equations:

$$\delta p = c\,(1 - p)^2\,(p + d)$$
$$\text{and } \delta x = c\,(1 - x)^2\,(x + d)$$

when δx is required to change sign at a certain point along the curve; when δx does not change sign the second equation can be provided by either: differentiating the expression for δx and equating the derivative to 0, the desired value of x for δx maximum can then be substituted and "d" found from:

$$d = \frac{2}{3}\,\frac{(x^2 + x + 1)}{(x + 4)}$$

or, using the expression for the centroid of $\delta \phi$, evaluated as shown in the paper, giving "h" the approximate desired value.

In order to put a check on the fairness of the new curves thus defined by the δx equations the appropriate limits were found as shown in the paper and compared with the $\delta \phi$ values corresponding to different bilge radii.

Sets of curves of δx were actually plotted for the $0 \cdot 75\ C_b$ parent form variations by the method shown in section 7 of the paper. Each set represented bilge radii of different values for one LCB position. There were 3 sets; for LCB at 1 per cent, $1 \cdot 5$ per cent, and 2 per cent forward of 'midships. This provided a ready means of constructing an area curve for any variation of parent form, bearing in mind that each of the ordinates had to be first multiplied by the new midship section area coefficient C_m.

The equation of the general form $x = x^n\,(1 - x^m)$ mentioned in section 8 of the paper was found useful in adjusting sections of the entrance and run to suit the new shape of midship section. One of the indices has to be given an arbitrary value by trial and error and the other can be found by again differentiating the expression for δx and equating the derivative to 0. The expression for x for δx maximum is then obtained, viz.:

$$x = m\sqrt{\frac{n + m}{n}}$$

Hence "n" can be found if "m" is already chosen for a desired concentration of increase or reduction of midship section area coefficient represented by δx maximum. Limiting values for δC_m can be determined in the same way as before for $\delta \phi$, but the process is more complicated.

Author's Reply

I am glad to have Mr. Kent's remarks and to know that an experimenter of his wide experience considers the methods of form variation given in the paper suitable for the intended purpose. It is agreed that in varying proportions and fineness care should be taken to verify that limiting slopes of water-lines, such as angle of entrance, etc., are within the usually accepted limits for good resistance performance. This applies particularly of course to the development of new designs in a design office, and in this connection reference would have to be made to published data on ship resistance for guidance, as this aspect is outside the scope of the paper. As regards applications of the methods to methodical series of resistance experiments, as Mr. Kent says, one of the objects would be to throw more light on such items as the critical limits of water-line slopes, displacement-length ratio, etc., and other geometrical features which are known to influence the resistance. In this connection I should like to endorse Mr. Kent's views on the importance of carrying out such work and, what is equally important, the discovery of the underlying reasons which cause such limits to be critical as far as resistance is concerned.

With regard to Mr. Tothill's remarks, I was interested to see his suggestion that the methods given in the paper

could be equally well applied to the curve of waterplane areas to vary the vertical prismatic coefficient and VCB in a consistent manner. Such an application had occurred to me in the early stages, although I had not given the matter very serious consideration. As far as I know, vertical distribution of displacement appears to have attracted little or no attention in connection with systematic series, and I am obliged to Mr. Tothill for raising the matter. Apart from possible effects on smooth water resistance performance, the vertical distribution of displacement has a bearing on the relation between ballast and load draught. From the point of view of seaworthiness in the ballast condition the necessity for having an adequate ballast draught has come into prominence of late, and in this connection it might be worth while investigating the possibility of increasing this draught by vertical redistribution of displacement.

With reference to the restriction mentioned by Mr. Tothill that the methods of variation cannot be applied to a vessel with a raked keel without buckling the keel-line, I think this difficulty could be conveniently overcome by executing the transfer of sections along the line of the raked keel rather than along the arbitrary horizontal base line. For this purpose, of course, the ship's form would have to be referred to an equivalent waterline parallel to the keel.

Referring to Mr. Tothill's remarks on Figs. 10 and 11 and their resemblance to the variations used in Taylor's Series, as far as I can ascertain, Taylor's area curves are mathematical in nature and correspond to 5th order polynomials in which the coefficients of the terms are varied to suit the required degree of fullness, etc. In that case of course the variations are not identical with those used in the present paper, although the curves for varying degrees of fullness may appear to be somewhat similar.

Mr. Benson's concurrence in the application of the processes given in the paper is very welcome, and I was interested to see that he has investigated the fairness of the shoulder of the derived curve in a few cases. With regard to the radius of curvature at the shoulder it is interesting to consider the relation between this in the parent and derived forms. From the relations given in section iii(c) of Appendix II it can be shown that

$$R' = R\left[1 + c\,(1 - 2p - d)\right]^2$$

where R and R′ are the radii of curvature at the shoulder in the parent and derived curves respectively, c, p and d being as defined in the paper. As c varies with $\delta\phi$ it will be seen, therefore, that in the derived form the radius of curvature at the shoulder of the area curve is either greater or less than that in the parent according to whether the prismatic coefficient is increased or decreased. Another point is that if there is no curvature at the shoulder (i.e. R infinite) in the parent form there will likewise be no curvature in the derived forms.

Concerning Mr. Benson's concluding remarks on the relative accuracy of the exact value and first approximation to the lever "h," the values of both terms in equation (3) have been worked out as requested for the particular case he mentions, viz. the entrance area curve shown in Fig. 11 when $\delta\phi_e$ is 15 per cent of ϕ_e. When evaluated this is given by

$$h = 0{\cdot}5519 + 0{\cdot}0641 = 0{\cdot}616$$

It will be noted that the second term is of the order of 10 per cent of the first and in this particular case can hardly be regarded as negligible. Of course a change $\delta\phi_e$ equal to 15 per cent is relatively large and should be regarded as outside the "moderate change in fineness" to which the first approximation was intended to apply. In such cases it would be advisable to use the exact levers for "h" where practicable. I should also like to point out that the area curve in Fig. 11 is a curve of entrance only and is not associated with its original parallel middle body, which tends to exaggerate the effect. Nevertheless, it is interesting to consider the effect of using the exact lever "h" rather than the approximate value on the position of the LCB of the form to which this entrance curve belongs, which after all is the final test of the application. The form in question is that used in the "Worked Examples" in Appendix I, where it will be seen that the LCB of the parent curve is 0·985 per cent of LBP forward of midships. Increasing the fullness of the entrance by 15 per cent in the same manner as above and using the first approximation to the lever "h" brings the LCB to 2·57 per cent of LPB forward of midships. Using the exact value of the lever moves it only a relatively small distance further forward to 2·69 per cent of LBP in spite of the relatively large change in fullness. The comparative effect of using the exact and approximate levers is also discussed in Example B of Appendix I.

Mr. Bocler's remarks from the ship design point of view are much appreciated, and I am glad to have his confirmation that the methods of form variation will be of service in the design office as well as for the development of forms for methodical series of resistance experiments. In particular the methods might prove useful in preparing preliminary body sections for new designs and in eliminating trial and error should save time and labour. The more general methods will enable control to be kept over the position and extent of parallel middle body if required, which was not possible in the well-known "one minus prismatic" and the "swinging" methods of variation. As Mr. Bocler rightly points out, however, the characteristics of the derived forms should be considered on their own merits before final acceptance.

Mr. Corin also refers to the necessity of confirming that the slopes of water-lines, etc., in derived forms are within practical bounds and more particularly where relatively large changes are made in the basic particulars. With regard to the position of the point of inflexion of the area curve in the forebody, possibly relations could be developed which would enable this to be controlled, but I feel they might prove to be rather unwieldy. As will be seen from Figs. 10 and 11, as the entrance curve is filled out by the methods described in the paper the point of inflection moves forward and vice versa.

It is true, as Mr. Corin points out, that in lifting new body sections directly from the half-breadth plan of the

basis ship, the bilge radius would tend to become slightly distorted if a change in beam-draught ratio is made. In practice, however, this is likely to be very small, and if it is desired to maintain a circular bilge this could no doubt be effected with very little fairing.

With regard to Dr. Weinblum's remarks it is noted that, except in the case of the "one minus prismatic" variation, he suggests varying the fullness of area curves by additions to the ordinates rather than displacing the ordinates in a fore and aft direction. This would appear to be natural enough in dealing with mathematical parent curves of the form $y = f(x)$, but experience shows that in the systematic variation of non-mathematical curves as generally used in practice, it is much simpler to adhere to the fore and aft shift of ordinates and, moreover, it is much more useful from the practical point of view. As a matter of interest, in first tackling this problem, variation in fullness was attempted by systematic mathematical additions to the ordinates of ships' area curves, but difficulty was always experienced in way of the shoulder because at some unpredictable limit there was always the danger that the derived curve would come outside of the line of maximum ordinate. By making the transformation by fore and aft shift of ordinates, however, this difficulty was completely overcome and, as shown in the paper, derived curves of good shape were obtained over a reasonably wide range within quite definite and easily predictable limits. Fore and aft shift of ordinates also lends itself very readily to systematic variation of the extent of parallel middle body which was one of the principle variables considered in the paper and, as already mentioned, is much more useful from the practical point of view. For example, as described in detail in section 7 of the paper, the water-line offsets of the displacement sections for the derived form can be lifted directly from the half-breadth plan of the basis ship without the necessity for first drawing out either the area curves or water-lines. This should be of assistance in preparing preliminary body sections for new designs. If the area curve were varied by additions to the ordinates, the procedure would be more indirect in that both parent and derived area curves would have to be drawn out and the necessary fore and aft shifts scaled off at each section. Further, the accuracy of this procedure might be questionable in some cases unless the curves were drawn on an abnormally and probably inconveniently large scale.

In view of the above remarks, therefore, I cannot agree that variations to the ordinates of areas curves keep their merits when the line is "empirical" and only given graphically as opposed to a purely mathematical curve. With regard to the remark that it is always possible to approximate to an empirical line within the required accuracy by simple functions such as polynomials, I have known cases where Taylor's polynomials were hardly sufficient in this respect.

In connection with the above argument it is interesting to reflect that the well-known and well-tried "swinging" variation to effect changes in LCB position is also essentially a "fore and aft" variation of form.

To come now to Dr. Weinblum's remarks on the apparent habit of naval architects of developing special kinds of mathematics peculiar to the profession and as to whether it is more mathematically orthodox to deal with one variable than another. I feel that this is very much a matter of opinion and would appear to be irrelevant in the present case. If experience shows that a certain approach gives the most straightforward and practically useful solutions, then I should say that the end justifies the means, that is, assuming that the particular approach is indeed unorthodox. No doubt this philosophy is not entirely unfamiliar even to pure mathematicians.

I gather from Dr. Weinblum's concluding remarks that he is inclined to favour the application of mathematical forms to methodical series of resistance experiments, and I agree that there is much to be said for this for fundamental investigations in connection with the calculation of resistance, oscillations in a seaway, etc. As far as actual ships are concerned, however, such forms appear to be of little interest to shipbuilders at the moment.

Dr. Hughes's remarks refer particularly to the longitudinal distribution of the added area over the length of the entrance (or run) which is discussed briefly in section 8 of the paper. With regard to the particular point he raises, i.e. the desirability of controlling the distribution of area between the shoulder and the half-length of the entrance, I should like to point out that, as mentioned in section 8, the distribution over the entrance could be controlled to almost any extent by using the methods given in the paper provided a certain amount of parallel middle body is introduced in conjunction with the change in fullness. This is covered by the general case dealt with in section 5, and if it is desired to effect a maximum shift δx at any fraction "s," say, of the length of entrance (measured from the shoulder), it can be shown that the necessary alteration in parallel middle body δp is given by

$$\delta p = \frac{\delta \phi \; (1-p)}{\dfrac{A}{(1-p)\,(1-2\,s)} + (1-\phi)}$$

The expressions for δx referred to by Dr. Hughes would enable the distribution of added area over the entrance to be controlled without altering the extent of the parallel middle body and might prove useful for special applications. Such variations were considered in the early stages of this work, and as a matter of interest the necessary relations were worked out for one of the variations he suggests, viz. $x = c \,.\, x\,(1-x)^2$ where the maximum shift x occurs at $x = \frac{1}{3}$. For the simple case of an entrance not associated with parallel middle body the necessary relations are as follows:—

$$\delta x = \frac{\delta \phi \,.\, x\,(1-x)^2}{\phi\,(1 - 4\,\bar{x} + 3\,k^2)}$$

and $$h = \frac{2\,\bar{x} - 6\,k^2 + 4\,r^3}{1 - 4\,\bar{x} + 3\,k^2} \quad \text{(first approximation)}$$

Varying the distribution of added area over the entrance in this manner, however, always involves higher powers of x than the simple form $x = c \,.\, x(1 - x)$ and in general the relations for δx and h, and particularly the latter, tend to become rather unwieldy. It was therefore decided to adhere to the simple form referred to for general use in conjunction with variation in the extent of parallel middle body.

I agree with Dr. Hughes that the generalized variation he suggests of the form $x = c \,.\, x(1 - x)^n$ is preferable to that mentioned in Section 8 of the paper, viz. $x = c\, x^n (1 - x^m)$. Actually the latter should have read $c \,.\, x^n (1 - x)^m$ of which Dr. Hughes's relation is a particular case, and I am obliged to him for having brought it to my notice. The position of the maximum shift δx would then become

$$x = \frac{n}{m + n}$$

It is gratifying to see that Mr. Weyndling has found so much of interest in the paper and to note that he has applied the methods in a design office in connection with the correlation of existing ship forms with the corresponding experiment tank resistance data.

Mr. Weyndling's approach to the above problem appears to have been to use the methods of derivation given in the paper to express analytically the relation between a family of area curves for a methodical series in terms of the parent form. These variations have then been applied to other forms for which tank resistance data was available with a view to extending the application of the findings of the methodical series to such forms.

In the particular B.S.R.A. series to which he refers the derivation of one form from another is more involved than a straightforward transference of sections in that one of the primary variables was the bilge radius. As this radius was increased the displacement lost amidships was made up by filling out the ends, but at the same time maintaining the original load water-plane. It is interesting to note that in fitting in the analytical curves Mr. Weyndling has used a more general type of relation of the form $\delta x = c(1 - x)^m (x + d)$, which is similar to that referred to by Dr. Hughes except that it enables the parallel middle body to be altered at the same time. In the circumstances remarks will apply to some extent as for Dr. Hughes's contribution. In this I showed that almost any distribution of added area over the entrance could be achieved by using the general case of Section 5 provided the appropriate amount of parallel middle body was introduced in making the change in fullness. Probably the actual differences in parallel middle body between the different area curves in the series were not appropriate to that which would give the desired distribution of added area, and this no doubt explains the use of the particular variation mentioned. Of course one would expect this inverse process of developing analytical expressions to relate existing families of area curves to be rather exacting and can be regarded as a special application for which the highly generalized relations discussed briefly in section 8 are particularly suitable. I should have expected, however, that, if the simpler variation $\delta x = c(1 - x)(x + d)$ of section 5 had been used with the appropriate change of parallel middle body to give the closest approximation to the required distribution of area, then any differences at the shoulder would have been small and probably negligible from the practical point of view.

In conclusion I should like to take the opportunity of thanking all contributors for having taken part in the discussion. It is apparent that there has been a need for information on the systematic derivation of ship forms, and the expression of views and experiences has added effectively to the value of the paper.